A MEASUREMENT OF THE BRANCING FRACTION OF THE SEMILEPTONIC DECAY FROM THE $D^+$ MESON TO THE $\bar{K}(892)^*$ MESON

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ABSTRACT

Using 8.78 fb$^{-1}$ of CLEO II.V data and 4.75 fb$^{-1}$ of CLEO II data, I have measured the ratios of the branching fractions $R_e^+ = \frac{B(D^+ \to \bar{K}^*0 e^+\nu_e)}{B(D^+ \to \bar{K}^*0 \mu^+\nu_\mu)}$, $R_\mu^+ = \frac{B(D^+ \to \bar{K}^*0 \mu^+\nu_\mu)}{B(D^+ \to \bar{K}^*0 \pi^+\pi^-)}$ and the combined branching fraction ratio $R_t^+ = \frac{B(D^+ \to \bar{K}^*0 t^+\nu_t)}{B(D^+ \to \bar{K}^*0 \pi^+\pi^-)}$. I have found $R_e^+ = 0.74 \pm 0.04 (\text{stat.}) \pm 0.05 (\text{sys.})$, $R_\mu^+ = 0.72 \pm 0.10 (\text{stat.}) \pm 0.05 (\text{sys.})$ and $R_t^+ = 0.74 \pm 0.04 (\text{stat.}) \pm 0.05 (\text{sys.})$. Using the known branching fraction $B(D^+ \to K^- \pi^+\pi^+)$, I have calculated the branching fractions of $D^+ \to \bar{K}^*0 e^+\nu_e$, $D^+ \to \bar{K}^*0 \mu^+\nu_\mu$ and $D^+ \to \bar{K}^*0 t^+\nu_t$ to be $B(D^+ \to \bar{K}^*0 e^+\nu_e) = (6.7 \pm 0.4 (\text{stat.}) \pm 0.5 (\text{sys.}) \pm 0.4)\%$, $B(D^+ \to \bar{K}^*0 \mu^+\nu_\mu) = (6.5 \pm 0.9 (\text{stat.}) \pm 0.5 (\text{sys.}) \pm 0.4)\%$ and $B(D^+ \to \bar{K}^*0 t^+\nu_t) = (6.7 \pm 0.4 (\text{stat.}) \pm 0.5 (\text{sys.}) \pm 0.4)\%$ where the third error is due to the error in $B(D^+ \to K^- \pi^+\pi^+)$. 
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sponding luminosities of CLEO II with respect to those of CLEO II.V. 
The number of events in the table means the number of events which 
have at least one $\pi^0$ found with the selection cuts in Table 3.1. The 
average numbers of $\pi^0$'s per event were calculated among those events.  
E.3 Comparison of number of the tracks between CLEO II.V and CLEO II. 
CLEO II.V numbers were scaled with the ratio of the corresponding 
luminosities of CLEO II.V with respect to those of CLEO II.  
E.4 Summary of the results of “$\nu$ reconstruction” method check with $D^+ \rightarrow$ 
$\bar{K}\pi^+\pi^+$ mode. In the table, $N$, $\epsilon$ and $N_e$ indicate raw yield, efficiency 
and efficiency-corrected yield, respectively.
CHAPTER 1
MOTIVATION

This thesis describes a study of the semileptonic decay of the charged $D$ meson to a neutral $K^*$. In this first chapter, I present the motivation of this study including a description of the theoretical background and an introduction of previous measurements of the branching fraction of the $D^+ \rightarrow K^{*0} l^+ \nu_l$ decay mode.

1.1 Standard Model

Elementary particle physics aims to answer the questions, “What are the elementary building blocks of matter?” and “How do these blocks interact with each other?” The current picture for this is the Standard Model, which successfully accounts for all known interactions except gravity. In this model, matter is composed of three generations of leptons and quarks, which are all fermions carrying spins of 1/2. The interactions among these fermions are mediated by the gauge bosons: photons for electromagnetic interactions, gluons for strong interactions and the $W^\pm$ and $Z^0$ for weak interactions. Each of these fermions and gauge bosons has a corresponding antiparticle which has the same mass but the opposite charge. The neutral gauge bosons, photons, gluons and $Z^0$ are their own antiparticles. Some of the physical properties of these particles are summarized in Tables 1.1, 1.2 and 1.3.

The electromagnetic interaction, mediated by photons, is the most familiar in daily life. The electromagnetic interaction is responsible for binding electrons and nuclei and holding atoms together in larger structures. Quantum electrodynamics (QED) is the theory governing this interaction. Since photons are massless, the electromagnetic force has an infinite range.

The strong interaction, governed by quantum chromodynamics (QCD), is mediated by gluons. Because of the “running” coupling constant which varies its value as
Table 1.1: Some physical properties of leptons.

<table>
<thead>
<tr>
<th>generation</th>
<th>particle</th>
<th>mass (MeV)</th>
<th>charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$\nu_e$</td>
<td>&lt; 0.003</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>$e$</td>
<td>0.511</td>
<td>-1</td>
</tr>
<tr>
<td>II</td>
<td>$\nu_\mu$</td>
<td>&lt; 0.19</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>$\mu$</td>
<td>105.7</td>
<td>-1</td>
</tr>
<tr>
<td>III</td>
<td>$\nu_\tau$</td>
<td>&lt; 18.2</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>$\tau$</td>
<td>1777.0</td>
<td>-1</td>
</tr>
</tbody>
</table>

Table 1.2: Some physical properties of quarks.

<table>
<thead>
<tr>
<th>generation</th>
<th>particle</th>
<th>mass (MeV)</th>
<th>charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$u$ (up)</td>
<td>1 - 5</td>
<td>2/3</td>
</tr>
<tr>
<td></td>
<td>$d$ (down)</td>
<td>3 - 9</td>
<td>-1/3</td>
</tr>
<tr>
<td>II</td>
<td>$c$ (charm)</td>
<td>1150 - 1350</td>
<td>2/3</td>
</tr>
<tr>
<td></td>
<td>$s$ (strange)</td>
<td>75 - 170</td>
<td>-1/3</td>
</tr>
<tr>
<td>III</td>
<td>$t$ (top)</td>
<td>174300</td>
<td>2/3</td>
</tr>
<tr>
<td></td>
<td>$b$ (bottom)</td>
<td>4000 - 4400</td>
<td>-1/3</td>
</tr>
</tbody>
</table>

Table 1.3: Some physical properties of gauge bosons.

<table>
<thead>
<tr>
<th>particle</th>
<th>interaction</th>
<th>mass (GeV)</th>
<th>charge</th>
<th>color</th>
<th>spin</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>Electromagnetic</td>
<td>0</td>
<td>0</td>
<td>None</td>
<td>1</td>
</tr>
<tr>
<td>$g$</td>
<td>Strong</td>
<td>0</td>
<td>0</td>
<td>Yes</td>
<td>1</td>
</tr>
<tr>
<td>$W^\pm$</td>
<td>Weak</td>
<td>80.419</td>
<td>$\pm$1</td>
<td>None</td>
<td>1</td>
</tr>
<tr>
<td>$Z^0$</td>
<td>Weak</td>
<td>91.188</td>
<td>0</td>
<td>None</td>
<td>1</td>
</tr>
</tbody>
</table>
a function of momentum transfer, strongly interacting particles are “asymptotically free” when they interact with large momentum transfers (and are close together). However, when they are far apart, the strong force becomes quite strong. In this way, quarks are confined in hadrons and all naturally occurring particles are “color singlets”. The energy between these two interaction domains, called the “QCD scale” $\Lambda_{\text{QCD}}$, is typically around 200 MeV.\footnote{Throughout this thesis, I use “natural units” where $\hbar = c = 1$. Mass, energy, and momentum are then measured in MeV or GeV.} Only quarks and gluons have colors and participate in this interaction.

The weak interaction was first discovered in radioactive decays of nuclei. In daily life, this interaction is hardly noticeable. Since the intermediate bosons for the weak interaction, $W^\pm$ and $Z^0$, are heavy, the range of this interaction is very short. All of the leptons and quarks take part in this interaction.

Even though the Standard Model has been successful, it has left many questions unanswered. How do fermions acquire masses? Why are masses of elementary particles so different? Are there more generations of leptons or quarks? Is it possible to unify the electroweak interaction and the strong interaction? Tremendous theoretical and experimental efforts are being made to answer these questions. Supersymmetric theory, for example, has been proposed as a potential extension of the Standard Model. Even if the Standard Model is not a complete theory, it will remain as an effective field theory since it is already a renormalizable relativistic quantum field theory.

### 1.2 Weak Charged-current Interaction and CP Violation

Within the Standard Model, the weak charged-current interaction is mediated by the $W$ bosons. Fig. 1.1 shows a vertex consisting of a $W$ boson coupling to two fermions $f_1$ and $f_2$. These fermions must belong to the same weak isodoublet. The weak...
Figure 1.1: A $W$ boson vertex with fermions $f_1$ and $f_2$. The fermions belong to the same weak isodoublet.

Isodoublets for leptons are

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}. \tag{1.1}$$

The weak eigenstate doublets of quarks are given by

$$\begin{pmatrix} u \\ d' \end{pmatrix}, \begin{pmatrix} c \\ s' \end{pmatrix}, \begin{pmatrix} t \\ b' \end{pmatrix}, \tag{1.2}$$

where $d', s'$ and $b'$ are not the same as the flavor eigenstates $d, s$ and $b$. They can instead be expressed as linear combinations of the flavor eigenstates:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}. \tag{1.3}$$

The matrix $V$ is known as the Cabibbo-Kobayashi-Maskawa (CKM) matrix, [1] and the square of the magnitude of $V_{ij}$ is the relative probability for a weak transition
to occur between quarks of flavor \( i \) an \( j \). Since off-diagonal elements of the CKM matrix are not zero, generation-changing transitions between quarks are allowed in weak charged-current interactions. The vertex of Fig. 1.1 is represented by

\[
\frac{-ig_w}{2\sqrt{2}}\gamma^\mu(1 - \gamma^5),
\]

(1.4)

where \( g_w \) is the “weak coupling constant”. The matrices \( \gamma^\mu \) and \( \gamma^\mu\gamma^5 \) represent vector and axial vector couplings, respectively, reflecting the \( V - A \) nature of the charged weak current. Since vectors and axial vectors behave differently under parity transformations, the charged weak current violates parity.

Since the CKM matrix is a \( 3 \times 3 \) unitary matrix, it is represented with four independent real parameters. In the Wolfenstein parameterization, [2] the CKM matrix is expressed in terms of \( \lambda, A, \rho \) and \( \eta \):

\[
\begin{pmatrix}
1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\
-\lambda & 1 - \lambda^2/2 & A\lambda^2 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix}.
\]

(1.5)

The complex component \( \eta \) in \( V_{ub} \) and \( V_{td} \) is responsible for CP violation in the Standard Model. Accordingly, all CP-violating amplitudes are proportional to \( A^2\eta\lambda^6 \). [2] Until recently, CP violation had been observed only in the neutral kaon system. Kobayashi and Maskawa introduced the third generation of quarks before it was discovered to explain this observation. A \( 2 \times 2 \) unitary matrix has only one real parameter and does not have a complex phase like the CKM matrix does. Recently BaBar [3] and Belle [4] announced the discovery of CP violation in the neutral \( B \) system which is consistent with the Standard Model expectation.

### 1.3 Decay Dynamics in \( D^+ \rightarrow \bar{K}^*_{0} l^+ \nu_l \)

The decay \( D^+ \rightarrow \bar{K}^*_{0} l^+ \nu_l \) will allow us to study the nature of the weak interaction and the effect of QCD. This is important because \( V_{cs} \) can be extracted from the branching fraction measurement combined with the form factor ratios and theoretical
models giving the $q^2$ dependence of the form factors and the normalization value. By using the heavy quark effective theory (refer to section 1.4) and $B$ decays, $|V_{cs}|$ can be extracted.

The decay dynamics of $D^+ \rightarrow \bar{K}^*0 l^+ \nu_l$ are represented by the interaction between the leptonic current of the $l^+$ and $\nu_l$ and the hadronic current composed of quarks and gluons. Fig. 1.2 shows its tree Feynman diagram\(^2\). This interaction is mediated by a virtual $W$ boson whose propagator is given by

$$-\frac{i(g_{\mu\nu} - \frac{q_{\mu} q_{\nu}}{M_W^2})}{q^2 - M_W^2},$$

(1.6)

where $q^\mu$ is the four-momentum carried by the virtual $W$ boson and $M_W$ is the mass of the $W$ boson. Since $q^2$ is so much smaller than $M_W^2$ in this process, equation 1.6 can be approximated as

$$\frac{ig_{\mu\nu}}{M_W^2}.$$  

(1.7)

Then, the decay amplitude is explicitly expressed by

$$\mathcal{A}(D^+ \rightarrow \bar{K}^*0 l^+ \nu_l) = \frac{G_F}{\sqrt{2}} V_{cs} \mathcal{L}^\mu \mathcal{H}_\mu,$$

(1.8)

where $G_F$ is the Fermi coupling constant in weak interactions given by $G_F = \frac{\sqrt{2}}{8} (\frac{g_\mu}{M_W})^2$ and $V_{cs}$ is the CKM matrix element for the weak transition from the $c$ to the $s$ quark. $\mathcal{L}^\mu$ and $\mathcal{H}_\mu$ represent the leptonic and hadronic current, respectively. Since the weak charged-current has the $V - A$ characteristics as implied by equation 1.4, the leptonic current $\mathcal{L}^\mu$ can be written as

$$\mathcal{L}^\mu = \bar{u}_l \gamma^\mu (1 - \gamma^5) v_\nu,$$

(1.9)

where $u_l$ and $v_\nu$ are Dirac spinors for the lepton and the neutrino. The hadronic current, however, is not easily expressed because of soft gluon exchanges between the quarks in the decay process. Since the only available four-vectors in the decay

---

\(^2\)A tree Feynman diagram does not contain any loops.
are $p_D$, $p_{K^*}$ and the polarization vector of the $K^*$, $\epsilon$, the hadronic current must be represented in terms of these four vectors. Utilizing the requirement that each term of the hadronic current must be linear in $\epsilon$, the hadronic current can be written as: [5]

$$
\mathcal{H}_\mu = (M_D + M_{K^*}) A_1(q^2) \epsilon_\mu - \frac{A_2(q^2)}{M_D + M_{K^*}} (\epsilon \cdot q)(p_D + p_{K^*})_\mu - \frac{A_3(q^2)}{M_D + M_{K^*}} (\epsilon \cdot q)(p_D - p_{K^*})_\mu - i \frac{2V(q^2)}{M_D + M_{K^*}} \epsilon_\nu \epsilon^{\mu \nu} p_D^\rho p_{K^*}^\rho,
$$

where $q^2$ is the mass-squared of the virtual $W$. $A_1$, $A_2$, $A_3$ and $V$ are called form factors and represent the axial and vector components of the hadronic current. Since $(p_D - p_{K^*})_\mu \mathcal{L}^\mu = 0$ in the limit of zero lepton mass, $A_3(q^2)$ is not important for electrons and muons.

### 1.4 Heavy Quark Effective Theory

Hadrons containing a “heavy” quark, such as $B$ or $D$ mesons, have an important symmetry. Here heavy means the mass of the quark is much larger than $\Lambda_{\text{QCD}}$. In such hadrons, the heavy quark and the light quark(s) interact by exchanging gluons with momentum transfer of the order of $\Lambda_{\text{QCD}}$. Since the Compton wavelength of the heavy quark is much less than $1/\Lambda_{\text{QCD}}$, the soft gluons do not have enough energy to
detect the mass or spin orientation of the heavy quark. As a result, the interaction between the light degree of freedom (light quark(s) and soft gluons) and the heavy quark is not disturbed by replacing the heavy quark with another heavy quark so long as their velocities are the same. This symmetry is called heavy quark symmetry (HQS). [6]

Corrections to HQS are typically of order $\Lambda_{QCD}/M_Q$, where $M_Q$ is the mass of the heavy quark. Heavy quark effective theory (HQET) [6, 7, 8] provides systematic methods for calculating these corrections. Using HQET, the form factor(s) of $c \rightarrow s$ decays can be related to that(those) of $b \rightarrow s$ transitions at the same $q^2$. As an example, the relations of form factors of $D^+ \rightarrow K^{*0}e^+\nu_e$ to those of $B^0 \rightarrow K^{*0}e^+e^-$ are given by [9]

\[
(a_+ + a_-)^{b \rightarrow s} = \left[ \frac{m_c}{m_b} \right]^{3/2} \left[ \frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right]^{-6/25} (a_+ + a_-)^{c \rightarrow s},
\]

\[
(a_+ - a_-)^{b \rightarrow s} = \left[ \frac{m_c}{m_b} \right]^{3/2} \left[ \frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right]^{-6/25} (a_+ - a_-)^{c \rightarrow s},
\]

\[
g^{b \rightarrow s} = \left[ \frac{m_c}{m_b} \right]^{3/2} \left[ \frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right]^{-6/25} g^{c \rightarrow s},
\]

\[
f^{b \rightarrow s} = \left[ \frac{m_c}{m_b} \right]^{3/2} \left[ \frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right]^{-6/25} f^{c \rightarrow s},
\]

(1.11)

where $m_b$ and $m_c$ are the masses of the $b$ and $c$ quark, respectively and $\alpha_s$ is the running coupling constant of the strong interaction. The relations between $f$, $g$, $a_+$ and $a_-$ and the form factors $A_1$, $A_2$, $A_3$ and $V$ are:

\[
A_1 = \frac{f}{M_{D^+} + M_{K^*}}, \quad A_2 = -(M_{D} + M_{K^*})a_+,
\]

\[
A_3 = \frac{g}{2M_{K^*}}(a_+ - a_-), \quad V = (M_{D} + M_{K^*})g.
\]

(1.12)

The form factors of $B \rightarrow K^*$ can, then in principle, be related to those of $B \rightarrow \rho$ using the light-quark SU(3)-flavor symmetry. Therefore, using HQET and the light-quark SU(3)-flavor symmetry, $|V_{ub}|$ can be extracted from the form factors of $D^+ \rightarrow K^{*0}l^+\nu_l$ and the experimental measurement of $\Gamma(B^+ \rightarrow \rho l^+\nu_l)$. 
1.5 The Goal of This Study

In this work, only the branching fraction of $D^+ \to K^{*0} l^+ \nu_l$ is measured. In other experiments, $^{[10]}$ form factor ratios $r_2 = \frac{A_2}{A_1}$ and $r_V = \frac{V}{A_1}$ of $D^+ \to K^{*0} l^+ \nu_l$ are obtained from angular correlations of the daughter particles. Combining these, I can calculate the form factors $A_1$, $A_2$ and $V$.

For this measurement, a data sample corresponding to about 14 fb$^{-1}$ of integrated luminosity was used. This data sample was collected at CLEO, an $e^+e^-$ collider running at the center-of-mass energies at the vicinity of the mass of the $\Upsilon(4S)$ resonance, a vector meson composed of $b$ and $\bar{b}$ quark pair (For details, see Chapter 2). The mass and full width of the $\Upsilon(4S)$ are 10.5800$\pm$0.0035 GeV and 14$\pm$5 MeV, respectively. $^{[11]}$

After the $e^+$ and $e^-$ collide, they decay to a virtual $\gamma$. Since the $\Upsilon(4S)$ mass is much smaller than the $Z^0$ boson mass, the contribution of virtual $Z^0$ boson is negligible. This virtual $\gamma$ decays and fermion pairs are produced. The cross-sections of these fermion pair productions at the center-of-mass energy at the $\Upsilon(4S)$ mass ($M_{\Upsilon(4S)}$) are shown in Table 1.4.$^3$ Events in which $e^+e^- \to q\bar{q}$, where $q$ stands for the $u, d, c$ and $s$ quark, are called continuum events. Most of time ($> 96\%$ $^{[11]}$) this $\Upsilon(4S)$ decays to $B^0\bar{B}^0$ or $B^+B^-$ pairs. These events are called $B\bar{B}$ events.

$D$ mesons are produced in two ways. In the first, $B$ mesons are produced by $\Upsilon(4S)$ decays and some of these $B$ mesons decay to $D$ mesons by the flavor-changing weak interaction. In the second, $D$ mesons are produced from $e^+e^- \to c\bar{c}$. In this work, the $D$ mesons produced from $e^+e^- \to c\bar{c}$ are studied (The reason for this is explained in Section 3.3).

1.6 Previous Measurements of $R_\ell$

Table 1.5 shows four branching fraction measurements for the decay $D^+ \to K^{*0} e^+ \nu_e$ normalized to the branching fraction of $D^+ \to K^- \pi^+ \pi^+$. The current world average $^{3}e^+e^- \to e^+e^-$ (bhabha scattering) is an exemption. In the bhabha scattering, there are two decay channels: the $s$- and $t$-channel. Other scattering processes, however, have only the $s$-channel. Theoretically the $t$-channel has an infinity cross-section when the crossing angle between $e^+$ and $e^-$ is zero. This is why the bhabha scattering has such a large cross-section in Table 1.4.
Table 1.4: The cross-sections of fermion pair productions from $e^+e^-$ collisions at the center-of-mass energy at $M_{T(4S)}$. These cross-sections are the effective cross-sections within the experimental acceptance. [12]

<table>
<thead>
<tr>
<th>$e^+e^-$</th>
<th>cross-section (nb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^+e^-$</td>
<td>$\sim 40$</td>
</tr>
<tr>
<td>$\mu^+\mu^-$</td>
<td>1.16</td>
</tr>
<tr>
<td>$\tau^+\tau^-$</td>
<td>0.94</td>
</tr>
<tr>
<td>$u\bar{u}$</td>
<td>1.39</td>
</tr>
<tr>
<td>$d\bar{d}$</td>
<td>0.35</td>
</tr>
<tr>
<td>$c\bar{c}$</td>
<td>1.30</td>
</tr>
<tr>
<td>$s\bar{s}$</td>
<td>0.35</td>
</tr>
<tr>
<td>$b\bar{b}$</td>
<td>1.05</td>
</tr>
</tbody>
</table>

Table 1.5: Measurements of $R^+_c = \frac{B(D^+\to \pi^0\nu_e\nu_e)}{B(D^+\to K^-\pi^+\pi^0)}$.

<table>
<thead>
<tr>
<th>value</th>
<th>experiment site</th>
<th>year</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.67 $\pm$ 0.09 $\pm$ 0.07</td>
<td>CLEO [13]</td>
<td>1993</td>
</tr>
<tr>
<td>0.62 $\pm$ 0.15 $\pm$ 0.09</td>
<td>OMEG [14]</td>
<td>1991</td>
</tr>
<tr>
<td>0.55 $\pm$ 0.08 $\pm$ 0.10</td>
<td>ARGUS [15]</td>
<td>1991</td>
</tr>
<tr>
<td>0.49 $\pm$ 0.04 $\pm$ 0.05</td>
<td>E691 [16]</td>
<td>1989</td>
</tr>
<tr>
<td>0.54 $\pm$ 0.05</td>
<td>PDG Average</td>
<td></td>
</tr>
</tbody>
</table>

value of $R^+_c = \frac{B(D^+\to \pi^0\nu_e\nu_e)}{B(D^+\to K^-\pi^+\pi^0)}$ is dominated by the E691 measurement in 1989. However, subsequent measurements have consistently yielded larger central values for $R^+_c$. The previous CLEO $R^+_c$ measurement was in 1993 with data sets corresponding to 1.7 fb$^{-1}$ of integrated luminosity and yielded a central value that was 1.5$\sigma$ larger than that obtained by E691. Since then CLEO has accumulated ten times more integrated luminosity, and I can measure this branching ratio with a precision which may resolve the apparent disagreement between E691 and later experiments.

There have been only two measurements for $R^+_\mu = \frac{B(D^+\to K^-\pi^+\pi^+\nu_e\nu_\mu)}{B(D^+\to K^-\pi^+\pi^0)}$. Furthermore, no $R^+_\mu$ measurements were made with the same data used for $R^+_c$ measurements. Therefore, there has been no direct check of the consistency between $R^+_c$ and $R^+_\mu$ with one data sample. Such a measurement addresses the universality of the weak coupling constant. Using the CLEO II.V and CLEO II data sets, it would be possible
Table 1.6: Measurements of $R_\mu^+ = \frac{B(D^+ \to K^{*0} \mu^+ \nu_\mu)}{B(D^+ \to K^- \pi^+ \pi^+)}$.

<table>
<thead>
<tr>
<th>value</th>
<th>experiment site</th>
<th>year</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.56 \pm 0.04 \pm 0.06$</td>
<td>E687 [17]</td>
<td>1993</td>
</tr>
<tr>
<td>$0.46 \pm 0.07 \pm 0.08$</td>
<td>E653 [18]</td>
<td>1992</td>
</tr>
<tr>
<td>$0.53 \pm 0.06$</td>
<td>PDG Average</td>
<td></td>
</tr>
</tbody>
</table>

to measure $R_e^+$ and $R_\mu^+$ with reasonable sensitivities. As a result, CLEO could provide such a consistency check with one data sample. Table 1.6 shows the two previous $R_\mu^+$ measurements.
CHAPTER 2
TOOLS

It is a great technical challenge to accelerate charged particles up to nearly the speed of light and collide them to produce new particles. It is also difficult to extract the experimental information that I want from the collision products flying away from the collision. Here I will briefly describe the tools I used in this work: the Cornell Electron Storage Ring (CESR), the CLEO detector, charged particle identification, photon identification, CLEO’s data flow and the Monte Carlo (MC) method to simulate collision data. [19]

2.1 The Cornell Electron Storage Ring

The Cornell Electron Storage Ring (CESR) is an $e^+e^-$ collider located at the Wilson Synchrotron Laboratory at Cornell University. [20] Its circumference is 768 meters. CESR is able to produce $e^+e^-$ collisions with center-of-mass energies between 9 and 12 GeV. First, electrons and positrons are accelerated in a linear accelerator (LINAC). Then, they are injected into the synchrotron and accelerated to 5 GeV. Once electrons and positrons reach their maximum energies in the synchrotron, they are transferred to the storage ring. Electrons and positrons travel in bunches at 390 thousand revolutions per second around the ring. There are nine approximately evenly spaced bunch trains for electrons and positrons, respectively, and each train contains two bunches of electrons or positrons. These eighteen positron and electron bunches travel in the opposite direction and collide only at the interaction point in the CLEO detector. To avoid parasitic collisions, electrostatic separators are used.

- The Linear Accelerator
  Electrons are emitted from a heated filament and accelerated by a microwave electric field in a 30 meter long vacuum pipe. Positrons cannot be produced
Figure 2.1: An overview of CESR. The electron and positron bunches were not drawn to scale.
in the same way as electrons. They are produced when about 140 MeV electrons are made to smash into a tungsten plate at an intermediate point in the LINAC. Electrons, positrons and X-rays emerge from these collisions. Positrons are selected by Radio-frequency (RF) cavities and quadrupole magnets. While positrons are accelerated, electrons are decelerated because they have opposite charges. As a result, when positrons are focused by quadrupole magnets, electrons are lost. These positrons are accelerated in the remaining parts of the LINAC. Before electrons and positrons are injected into the synchrotron, their energies are as large as 200 MeV for positrons and 300 MeV for electrons, respectively.

- The Synchrotron

Electrons or positrons from the LINAC are accelerated in the synchrotron to the energy at which they can be stored efficiently in CESR, typically 5 GeV. The synchrotron consists of four three-meter long linear accelerators and 192 three-meter long sections of bending and focusing magnets. After a bunch of electrons or positrons has made 4000 turns around the ring, it reaches its maximum energy and is injected into the storage ring. The entire acceleration cycle is repeated for electrons and positrons until the required beam currents are built up in the storage ring. To reduce collisions between electrons or positrons and air molecules, a good quality of vacuum must be maintained in the synchrotron ring.

- The Storage Ring (CESR)

The storage ring uses magnets similar to those in the synchrotron. Since electrons or positrons must stay in the storage ring for hours, the quality of the vacuum in the ring and the precision of the magnet system is much higher here than in the synchrotron. Electrons and positrons emit synchrotron radiation while they travel around the ring and lose some of their energies. This lost energy is restored using RF cavities running at 500 MHz.
CESR has operated on and near the mass of the $\Upsilon(4S)$ resonance. The luminosity $\mathcal{L}$ is given by

$$\mathcal{L} = fn \frac{N_{e^+}N_{e^-}}{A},$$

(2.1)

where $f$ is the revolution frequency, $n$ is the number of bunches in each of the $e^+e^-$ beams, $N_{e^+}$ and $N_{e^-}$ are the numbers of electrons and positrons in each bunch, and $A$ is the cross-section of the beams. The number of events in a certain decay mode $N$ depends on the integrated luminosity of the data sample $\mathcal{L}$ because $N$ is given by $N = \sigma \mathcal{L}$ where $\sigma$, the cross-section of the decay mode, is constant at a given energy. Fig. 2.2 shows the history of the CESR annual integrated luminosity. The little annual integrated luminosities in 1995 and in 1999 were due to long shutdowns while the CLEO detector was upgraded. The CLEO II detector was upgraded in 1995 and the resulting detector was called CLEO II.V. In 1999, the CLEO II.V detector was upgraded and called CLEO III. The instantaneous luminosity of CESR was increased to $10^{33}\text{cm}^{-2}\text{sec}^{-1}$ level after the CLEO III upgrade.

### 2.2 The CLEO Detector

CLEO is a solenoidal detector that measures the momenta of charged particles and the energies of electromagnetic showers arising from electrons and photons with excellent resolution and efficiency. It also identifies electrons and muons with high efficiency and small contamination. Low energy hadrons can also be identified using their specific ionization energy losses ($dE/dx$) and time-of-flight measurements. The main components of the CLEO detector consist of a charged particle tracking system, a time of flight detector, a CsI electromagnetic calorimeter and muon detectors. Fig. 2.3 shows a side view of the detector. The dimension of the detector is about 6 meters on a side. Electron-positron collisions take place at the center of the detector. When a particle travels from the interaction point radially outward, it will encounter detector components in the following order:

1. Tracking Devices

From inside to outside, the charged particle tracking devices in CLEO II in-
Figure 2.2: The annual integrated luminosity history of CESR.
Figure 2.3: A side view of CLEO II.V cross-section.
cluded a precision tracking layer detector (PT), a vertex detector (VD), and a main drift chamber (DR). They formed co-axial cylinders and operated inside a 1.5T magnetic field. The CLEO II.V detector used a silicon vertex detector (SVX) instead of the PT. The fundamental principle of these charged particle tracking devices is the same. All of them except the SVX have anode (sense) wires held at high positive voltage and grounded field wires or a field tube around each anode. One anode wire and surrounding field wires or a field tube form a unit cell for tracking. All of these devices except the SVX are filled with gases. When a charged particle passes through the gas, molecules are ionized. The liberated electrons are accelerated toward the anodes, while ions move toward the field wires or tubes. A chain of the secondary ionization is initiated by the drifting electron as it enters the region of high electric field around the anode wire. This creates current in the anode wires which is then amplified by electronics at the wires’ ends. The amplified signal is sent to two separate circuits to determine its arrival time and pulse height. These are used to determine the distance of the particle trajectory to the anode wire and the ionization energy loss (dE/dx) of the charged particles.

Semiconductor material in the SVX plays the role of gases in the tracking chambers. The semiconductor material is installed as wafers. These wafers are partly p-type semiconductor and partly n-type, so that each wafer contains many thin and long p-n junctions. A reverse-bias voltage is applied across each junction creating a region which is depleted of charge carriers. When a charged particle passes through the wafer and loses its energy, about 20,000 electron-hole pairs are created in the depletion region. These electron-hole pairs are drifted to metal strips on the surface of the wafer by the applied electric field. The detection of this charge on a certain strip indicates that a charged particle passed through nearby. The position resolution of the SVX is about 30μm, much superior to the 150μm resolution of the gas-based tracking chambers.

The PT, VD and DR of the CLEO II detector are together called the central tracking detectors (CD). The CD of CLEO II.V implies a combination of the
SVX, VD and DR.

2. Time-of-Flight
The time of flight detector (ToF) is composed of plastic scintillators, light guides and photomultiplier tubes. The barrel ToF surrounding the DR is composed of 64 plastic scintillators. Two endcap ToF detectors cover the two sides of the DR and each of them consists of 28 plastic scintillators. Each of the scintillators in the barrel ToF has two light guides at its two ends and both of these light guides are connected to their own photomultiplier tubes. A plastic scintillator in the endcap ToF is connected to a light guide and a photo-multiplier tube. The light produced when a charged particle passes through the scintillator is transported through the light guide to the photomultiplier tube which then amplifies the signal. This signal starts a clock and this clock is stopped at the the end of the gate. Since the interval between two successive gates is synchronized to the known interval between two successive beam crossings, the time of flight of a charged particle from the interaction point to the time of flight detector is calculated.

3. Crystal Calorimeter
The crystal calorimeter (CC) consists of 7800 thallium-doped cesium iodide (CsI) crystals. This calorimeter is finely segmented to obtain good position resolution for calorimeter showers. The barrel CC surrounds the barrel ToF and is composed of 6144 CsI crystals, arranged in groups of 48 along the beam axis and 128 azimuthally. Two symmetric endcap CC’s cover the two endcap ToF’s and each of the endcap CC’s has 828 CsI crystals.

When electrons or photons pass through the CC, they lose most of their energy by pair production and bremsstrahlung. However muons and hadrons deposit little of their energy in the CC. This allows precise energy measurements of showers produced by electrons and photons in the CC as well as identification of electrons (see subsection 2.3.1 for details).

4. Superconducting Coil
A coil of superconducting wire provides a uniform 1.5 T magnetic field parallel to
the beam line over its full volume. This magnetic field causes charged particles
to move along helical trajectories and makes momentum measurements possible
from curvatures of the particles.

5. Muon Chamber
The outermost detectors in CLEO are the muon detectors (MU). They have
three layers of plastic streamer counters which operate in a manner similar to
the DR. The first layer is outside of the magnet return yoke and is followed by
two more layers of 36 cm thick iron and detectors. The magnet return yoke
serves not only as a return path for the magnetic flux but also as a hadron
absorber. As a consequence, most of particles reaching the first layer of the MU
are muons. Each cell of the MU has an anode wire and is filled with argon-ethane
gas similar to that in the tracking chambers\(^1\). The basic operation principle of
the MU is the same as that of the other tracking devices.

### 2.3 Charged Particle Identification

Charged particle identification can be subdivided into lepton \((e^\pm\) and \(\mu^\pm\)) identification and hadron \((\pi^\pm, K^\pm, p \text{ or } \bar{p})\) identification.

#### 2.3.1 Lepton Identification

For electron identification, information from the CD, the CC, and the ToF is used to
estimate a “likelihood” for a track to be an electron. This likelihood is composed of
several terms but the foremost among these is \(E/p\), the ratio of the energy deposited
in the CC by the track and its momentum, measured using the CD. Electrons deposit
most of their energies into the CC but charged hadrons or \(\mu\) leave only a part of their
energies. As a result, \(E/p\) for electrons is approximately unity but that for charged
hadrons or muons is less than unity. Fig. 2.4 shows \(E/p\) distributions of electron
candidates selected using the electron identification criteria in section 3.2 (solid line)

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\(^1\)The CLEO II/IV detector has helium-propane gas in its DR and MU.
and of those tracks rejected by the criteria (dashed line). The characteristics of showers in the CC depend upon the particles that created them. Electrons (positrons) and photons electromagnetically interact with the CC material and more electrons (positrons) and photons are produced by pair production and bremsstrahlung. These electron (positron) and photon showers lose all of their energies by ionization. However, hadrons and muons deposit only part of their energies in the CC. This is why electron candidates have a sharp $E/p$ peak around 1.0 but non-electron tracks do not. Other important parameters of the likelihood include $dE/dx$ measured in the DR (refer to Fig. 2.5), the matching distance between the track and the center of the closest shower, shower lateral shape, and the time-of-flight information from the ToF. The values of these parameters are compared to distributions obtained using embedded radiative bhabha events and $T(1S)$ events to estimate the likelihood. The electron identification efficiency is excellent at CLEO (see APPENDIX D). The probability for charged tracks other than electrons to be identified as electrons is very low, less than a few tenths of a percent when typical electron identification criteria are applied (see the electron identification criteria in section 3.2).

Muon identification utilizes the ability of muons to penetrate materials. All tracks having matching hits in the CD and the MU are selected as muon candidates and characterized by a variable called $\text{dpthmu}$. This is the amount of iron the track penetrated in terms of nuclear interaction lengths. Since normal incident muons have to cross three interaction lengths of iron to reach the first layer of the MU, they have $\text{dpthmu} \geq 3$. Those reaching the second layer will have a value of $\text{dpthmu} \geq 5$, and those reaching the third will have $\text{dpthmu} \geq 7$. The efficiency of muon identification at CLEO is not as good as that for electrons. For example, the MU is completely inefficient for muons whose momenta are less than 1.4 GeV when the muon selection criterion of $\text{dpthmu} > 5$ is used. The solid angle coverages of the MU are $0.85 \times 4\pi$, $0.82 \times 4\pi$ and $0.79 \times 4\pi$ when the requirements of $\text{dpthmu} \geq 3$, $\text{dpthmu} \geq 5$ and $\text{dpthmu} \geq 7$ are used, respectively. Furthermore the fake rate of muons (the probability for non-muon particles to be identified as muons) is much higher than that of electrons. About $1.5 \pm 0.5\%$ of hadrons with momentum greater than 1.5 GeV are identified as muons when $\text{dpthmu} > 5$ is required.
Figure 2.4: $E/p$ distributions of electron candidates and non-electron candidates for the CLEO II.V data. These distributions were obtained using the electron identification criteria described in section 3.2.
2.3.2 Hadron Identification

CLEO uses the DR and the ToF for charged-hadron identification. Since the mass ($M$), momentum ($p$), and velocity ($\beta$) of a particle are related by $p = M\beta\gamma$, where $\gamma = 1/\sqrt{1 - \beta^2}$, I can identify particles (knowing $M$) by measuring $p$ and $\beta$. Since the $dE/dx$ measured in the DR is proportional to $1/\beta^2$, it is useful in identifying relatively slow hadrons. The $dE/dx$ of a track is determined by measuring the charge collected on the DR wires along the path of the particle. The charge on each wire is then converted to energy deposited in each drift cell. To ensure good $dE/dx$ resolution, $dE/dx$ information is used for tracks that have hits in at least eleven drift cells in my analysis. Fig. 2.5 shows the $dE/dx$ distributions for the various species of particles with respect to their momenta.

The ToF measures the time for a particle to reach the ToF counter ($T$). Since $T = L/\beta$, where $L$ is the flight distance, $T$ is proportional to $1/\beta$. The distributions of $1/\beta$’s for $e$, $\mu$, $\pi$, $K$ and $p$ are shown in Fig. 2.6.

As shown in the two distributions, it is almost impossible to distinguish charged $\pi$’s from charged $K$’s using $dE/dx$ or ToF information if their momenta are greater than 0.8 GeV. As a result, this hadron identification information was not used in this work (details are described in section 3.5).

2.4 Photon Identification

The energy and direction of photons are measured by the CC. Empirically obtained expressions for the energy and angular resolutions of the CC are:

$$\frac{\sigma_E}{E} [= \%] = \frac{0.35}{E^{0.75}} + 1.9 - 0.1 \ E,$$  \hspace{1cm} (2.2)

$$\sigma_\phi[\text{mrad}] = \frac{2.8}{\sqrt{E}} + 1.9, \hspace{1cm} (2.3)$$

and

$$\sigma_\theta[\text{mrad}] = 0.8 \ \sigma_\phi \ \sin\theta \hspace{1cm} (2.4)$$
Figure 2.5: $dE/dx$ distributions for $e$, $\mu$, $\pi$, $K$, $p$ and $d$. The means for the different species are shown with solid line.
Figure 2.6: The distributions of $1/\beta$'s for $e$, $\mu$, $\pi$, $K$ and $p$. 
for photons in the barrel and

\[ \frac{\sigma_E}{E} [\%] = \frac{0.26}{E} + 2.5, \]  

(2.5)

\[ \sigma_\phi [\text{mrad}] = \frac{3.7}{\sqrt{E}} + 7.3, \]  

(2.6)

and

\[ \sigma_\theta [\text{mrad}] = \frac{1.4}{\sqrt{E}} + 5.6 \]  

(2.7)

for photons in the endcaps. Here \( E \) is the energy of the photon in GeV.

### 2.5 Data Flow

While the detector is being read out, it cannot accept new events. Since CLEO operates at high luminosities, it is essential to trigger the readout only for interesting events. To do this, CLEO has three hardware trigger systems: LEVEL0, LEVEL1 and LEVEL2. These trigger systems choose interesting events based on the information from the tracking devices, the ToF, and the CC.

LEVEL0 uses information from the VD, the ToF and the CC. For triggering purposes, the barrel ToF and each of the endcap ToF’s are divided into 16 and 7 sectors by ganging four adjacent scintillators. Similarly, the CC is also segmented into several sectors. The barrel CC has 16 sectors (2 sectors along the beam axis and 8 sectors azimuthally) and each of the endcap CC’s is divided into 8 sectors. The hit topologies over these ToF and CC sectors are used as the input of LEVEL0. Using the charge collected on the anode wires of the VD, charged particle multiplicity and momenta are found by a crude tracking processor called the Track Segment Processor. This information from the VD is incorporated into the LEVEL0 decision. Since the CC trigger is slow compared to the ToF or VD trigger\(^2\), all LEVEL0 triggers are separated into two categories:(i) events passing the the ToF and VD triggers, and (ii) events satisfying only the CC trigger such as \( e^+e^- \rightarrow \gamma\gamma \) or low multiplicity and

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\(^2\)The decision time of the CC trigger is approximately 3 \( \mu s \), and those of the VD and ToF triggers are 280 ns.
low transverse-momentum events. After final trigger selection, only 2.5% of all events belong to the second category. [21]

Since LEVEL0 pre-selects events, LEVEL1 has enough time to use more information from the tracking devices, the ToF, and the CC. LEVEL2 uses even more information from the tracking devices, and makes more sophisticated decisions. The event rates and dead times of these three trigger levels at the instantaneous luminosity of $2.0 \cdot 10^{32} \text{cm}^{-2}\text{s}^{-1}$ are 10 kHz, 100 Hz and 25 Hz and 2.5%, 0.6% and 5.0% for LEVEL0, LEVEL1 and LEVEL2, respectively.

Events passing the criteria of the three trigger systems have to pass another hoop, LEVEL3, [22] to be recorded. At the $\Upsilon(4S)$ center-of-mass energy, about 30% of outputs from LEVEL2 are useless events for physics analyses, i.e., events from interactions between the beams and the beam pipe walls or gas molecules in the beam pipe. Another 20% of the events are bhabha events. A software trigger LEVEL3 was designed to reject these events. LEVEL3 rejects about 45% of incoming LEVEL2 triggers while rejecting less than about 1% of events useful for physics analyses.

The surviving events are recorded on tapes. Then PASS2, an event reconstruction processor, reconstructs charged particles and electromagnetic showers, identifies particles, and classifies events into various categories, i.e., hadrons, bhabhas, muon pairs, etc. This information is used for data analyses.

### 2.6 Monte Carlo

In an analysis for a specific physics process, I need to know how the detector is response to the process of interest and differs from its response to other processes. To accomplish this, Monte Carlo (MC) simulations are used. The Monte Carlo simulation software is divided into two steps: physics process simulation and detector response simulation. The physics process simulation in CLEO is done by a program named QQ. QQ uses up-to-date knowledge of how quarks in $e^+e^- \rightarrow q\bar{q}$ reactions materialize as hadrons and how these hadrons decay. It also knows masses, charges, decay modes and branching ratios.

Event simulation requires modeling of the interaction between generated particles
and parts of the detector elements. This is done by a program called GEANT, [23] which contains detailed information about each part of the CLEO detector and how it interacts with particles. Through this simulation, I can estimate the detection efficiency of a particle with a certain momentum or the resolution of various measurements. Some of the essential properties of the detector like the position resolution of the tracking devices were extracted from data and used as inputs to GEANT. These results have been compared to data using well-understood events such as $e^+e^- \rightarrow e^+e^-$ or $\mu^+\mu^-$. The output of this simulation has the same format as that of real data. After reconstructing MC simulation events, I can analyze them as if they are real data.
CHAPTER 3
SEMILEPTONIC DECAY MODE ANALYSIS

3.1 Overview of Analysis Process

This analysis used CLEO II.V and CLEO II datasets whose integrated luminosities are about 8778 pb$^{-1}$ and 4746 pb$^{-1}$, respectively. The decay chain investigated was $D^{*+} \rightarrow D^+\pi^0$, $D^+ \rightarrow \bar{K}^*\ell^+\nu$, and $\bar{K}^* \rightarrow K^-\pi^+$. The decay $D^{*+} \rightarrow D^+\pi^0$ was used to obtain a cleaner sample of $D^+$ decays.

The following steps were taken to find the number of $D^+ \rightarrow \bar{K}^*\ell^+\nu$ decays:

1. Select continuum events and reject $B$ decay events.
2. Form $K\pi\ell$ combinations.
3. Reconstruct the $\nu$ using estimates of the $D$’s direction and a $D$ mass constraint.
4. Make and apply kinematic cuts to reduce backgrounds.
5. Fit the $K\pi$ mass distribution to find the number of $K^*$’s.
6. Fit the distribution of the mass difference between the $D^*$ and $D$ candidates to extract the number of signal events.

3.2 Signal Candidate Selection

The neutrino reconstruction method in this analysis assumes that the thrust axis of the event approximates the direction of the $D$ meson. This approach does not work well when events are not jetty$^1$. So events with small jettiness were rejected. Many of

$^1$In $B\bar{B}$ events at CLEO, $B$ mesons typically have momentum of 340 MeV. As a result, daughter tracks and showers of $B$ mesons are spherically isotropic. However, in continuum events, initial quarks have significant momentum. By this boost, their daughter tracks or showers form jets and the shape of such events becomes jetty.
the $D$ mesons arising from $B\bar{B}$ events were therefore rejected. In addition, $D$ mesons from $B\bar{B}$ events do not give statistical gain because the momenta of $D$ mesons in $B\bar{B}$ events are softer than those in continuum events and suffers from a lot of random combinations. To reject non-jetty events, R2GL$^2$ was required to be greater than 0.2.

To select well reconstructed tracks, each track was required to:

- pass close to the interaction point
- have $z$-axis information
- not be reconstructed with CD hits remaining after reconstruct all other tracks are found
- not be an additional track made by a curling particle
- be reconstructed successfully with all of the particle hypotheses, \textit{i.e.}, the $\pi$, $K$, $e$, $\mu$ and $p$ hypothesis
- have a momentum less than 6.0 GeV

To select good $\pi^0$'s, all $\gamma\gamma$ pairs for which $|M_{\gamma\gamma} - M_{\pi^0}| < 2.5\sigma$ \textsuperscript{3} were accepted. To identify leptons, the standard CLEO lepton identification requirements were used:

- R2ELEC $> 3.0$

- the momentum of an electron candidate is greater than or equal to 0.7 GeV

- $|\cos\theta| \leq 0.81$ where $z$-axis is the positron beam direction

The conditions to identify muons were:

- CD hits and the MU hits are matched well

- DPTHMU $\geq 5.0$

\textsuperscript{2}The second Fox-Wolfram moment, [24]

\textsuperscript{3}The resolution of $\pi^0$ mass distribution obtained using clean $\pi^0$ data samples as a function of $\pi^0$ momentum.
• momentum of a muon candidate is greater than or equal to 1.4 GeV when $|\cos \theta| \leq 0.61$, and greater than or equal to 1.9 GeV when $0.61 < |\cos \theta| \leq 0.81$.

Using these tracks, $\pi^0$'s and lepton candidates, I found a set of kinematic cuts to optimize my sensitivity. Since I obtain yield by fitting the $\delta m$ distribution, I need to maximize signal and minimize background in the region where the signal is expected to peak. Therefore I used events satisfying $\delta m < 0.15$ GeV to optimize other cuts. A set of kinematic cuts were tuned to obtain optimal signal squared over background ($S^2/N$). While $S^2/(S+N)$ should be used to obtain the best sensitivity, the background is dominant in this analysis so it is acceptable to use the background in the denominator instead of the sum of the signal and the background. QQ tagged$^4$ MC signal events were used to estimate the signal yield. About 140000 signal MC events were generated for each of the CLEO II.V and CLEO II data sets using the ISGW2 model to predict the matrix element and resulting electron spectra for semileptonic meson decays. [35] These signal MC samples were statistically independent of the signal samples used for efficiency calculations or the samples for the $\delta m$ signal shape estimation for the $\delta m$ fits to find the number of reconstructed signal events. For the background yield, about 30 million generic CLEO II.V continuum MC events and about 15 million generic CLEO II continuum MC events were used. In these continuum MC samples, the events containing the $D^+ \rightarrow \bar{K}^{*0}l^+ \nu_l$ decay mode were discarded using the QQ tagging method. The final optimized kinematic cuts are given in Table 3.1 with the global and the $\pi^0$ selection criteria.

3.3 Neutrino Reconstruction

To reconstruct the momentum of the $\nu$ in an event, two methods were used to obtain up to three values of $\bar{p}_\nu$. Fig. 3.1 shows how the first method works. It illustrates the momentum of $K\pi l$ system and how the thrust direction is assumed to represent the $D$ direction. Two possible values of $\bar{p}_\nu$ are also shown. Since the vector sum of $\bar{p}_{K\pi l}$ and $\bar{p}_\nu$ should equal $\bar{p}_D$, the heads of $\bar{p}_{\nu_1}$ and $\bar{p}_{\nu_2}$ should lie on the thrust direction.

$^4$The method using the generator-level decay information of a MC sample is called “QQ tagging”.
Table 3.1: Analysis cuts. In the $\pi^0$ selection raws, $\gamma$’s are the daughters of $\pi^0$.

<table>
<thead>
<tr>
<th>type</th>
<th>variable</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>global</td>
<td>KLASGL(^a)</td>
<td>$= 10$</td>
</tr>
<tr>
<td>global</td>
<td>NTRKCD(^b)</td>
<td>$\geq 3$</td>
</tr>
<tr>
<td>global</td>
<td>R2GL</td>
<td>$\geq 0.2$</td>
</tr>
<tr>
<td>$\pi^0$ selection</td>
<td>energy of $\gamma$</td>
<td>no match between $\gamma$’s and tracks at least one $\gamma$ has $</td>
</tr>
<tr>
<td>$\pi^0$ selection</td>
<td>energy of $\gamma$</td>
<td>$\geq 30$ MeV when its $</td>
</tr>
<tr>
<td>$\pi^0$ selection</td>
<td>$</td>
<td>M_{\gamma\pi} - M_{\pi^0}</td>
</tr>
<tr>
<td>kinematic</td>
<td>$</td>
<td>\vec{p}_{\pi^+}</td>
</tr>
<tr>
<td>kinematic</td>
<td>$</td>
<td>\vec{p}_{K^-}</td>
</tr>
<tr>
<td>kinematic</td>
<td>$</td>
<td>\vec{p}_{\pi^0}</td>
</tr>
<tr>
<td>kinematic</td>
<td>$</td>
<td>\vec{p}_{K\pi}</td>
</tr>
<tr>
<td>kinematic</td>
<td>$M_{K\pi}$</td>
<td>$1.2 - 1.8$ GeV</td>
</tr>
<tr>
<td>kinematic</td>
<td>$</td>
<td>\vec{p}_{K}\pi</td>
</tr>
</tbody>
</table>

\(^a\) Event class; < 10 means QED, $\gamma\gamma$, cosmic ray events, etc. 10 means hadronic events and 11 means beam gas events.

\(^b\) Number of charged tracks.
This locus satisfies \( M_{K\pi
u} = M_D \).

Figure 3.1: The locus with respect to the \( p_{K\pi} \) on which \( \vec{p}_D \) would lie and two estimations of \( \vec{p}_D \)'s when the direction of the \( \vec{p}_D \) is assumed to be the thrust direction of the event.

In addition, the invariant mass of \( K\pi\nu \) system should equal \( M_D \). This constraint corresponds to a locus (ellipsoid in three dimension) around \( p_{K\pi} \) in the figure. These two constraints generally lead to two solutions, also shown in the figure.

In continuum events, \(|\vec{p}_D|\) is typically about 2.5 GeV. As a consequence, the direction of the \( D \) meson in a 2-jet event would be very close to the thrust axis of the event. To study how good an approximation using the thrust axis to represent the \( D \) direction is, I used a signal MC sample. Fig. 3.2 (left) shows the angle between the thrust axis and the generated \( D \) direction. Its average is approximately 10°. If any of the final-state particles of an event, such as a \( \nu \), is missing, one would expect that the angle between the thrust of the event and the direction of the \( D \) would be larger. This is so in an event containing the decay \( D^+ \rightarrow \bar{K}^{*0}l^+\nu_l \), where neutrino is not detected. With a signal MC sample of \( D^+ \rightarrow \bar{K}^{*0}e^+\nu_e \), I quantified how much this missing momentum makes this angle larger than when no final particles are missing. The thrust axis of the event was recalculated including the generated \( \nu \) momentum and the angle between this axis and the generated direction of \( D \) was measured. Fig. 3.2 (right) shows that the improvement is less than 1°. As a consequence, the missing momentum of an event, which is the best estimate of the \( \nu \) momentum, was not included in the thrust estimation.

When the thrust direction of an event was used as the direction of the \( D \), sometimes there is no solution (i.e., the \( D \) direction vector and the ellipsoid have no intersection). In such cases, the \( D \) direction was manually moved within the plane.
Figure 3.2: Angle between the generated $D$ directions and the thrust calculated (left) without the generated $\vec{p}_\nu$ and (right) with the generated $\vec{p}_\nu$.

containing the thrust direction and $\vec{p}_{K\pi\ell}$ so that the new $D$ direction was tangential to the ellipsoid and gave one intersection point or solution.

The second method used the missing momentum of each event as an estimate of the $\nu$ momentum. For the missing momentum measurement, both hemispheres of the event were used. No additional cuts were made for this missing momentum measurement since applying additional cuts would reduce the statistics without appreciable gain in resolution.

Among these three $\nu$ momentum estimates, the one which gave the value of $\delta m$ closest to the known value, 0.141 GeV, [11] was chosen. In this solution selection process, if the $\nu$ momentum estimate from the missing momentum of an event gives a value of $M_{K\pi\ell}$ greater than $M_{D^+}$, 2.01 GeV, then the $\nu$ momentum estimate from the missing momentum was not considered as a solution. To quantify how well this selection method works, the frequency for this selection method to give the $\nu$ momentum closest to the generated $\nu$ momentum was counted with my signal MC sample and was found to be 75%. Among the three estimates, the one that was obtained with the thrust axis and gave smaller $|\vec{p}_D|$ was chosen in 65% of the accepted events. Fig. 3.3 shows the selected $|\vec{p}_D|$ type distribution. Because of the hardness
of the $|\vec{p}_{K\pi}|$ cut, $|\vec{p}_{K\pi}| > 2.0$ GeV, relative to the typical $D$ momentum, it is more likely that $\vec{p}_{D1}$ in Fig. 3.1, which has smaller $|\vec{p}_D|$ and makes $\vec{p}_\nu$ go against $\vec{p}_{K\pi}$ in the lab frame, is the right solution.

To estimate how much this $\nu$ reconstruction would help reducing statistical uncertainty, the width of the peak in the $\delta m$ histogram and that of the pseudo $\delta m$ histogram were compared, where $\delta m$ and pseudo $\delta m$ are defined as $M_{K\pi\nu\pi^0} - M_{K\pi\nu}$ and $M_{K\pi\pi^0} - M_{K\pi\pi}$, respectively. For reference, the previous CLEO analysis used pseudo $\delta m$ rather than $\delta m$. Fig. 3.4 shows fits of the peaks of $\delta m$ and pseudo $\delta m$ plots with single Gaussian function. Fig. 3.5 gives the comparison of $\delta m$ and pseudo $\delta m$ in the signal and in the background. In Fig. 3.5 (left), $\delta m$ and pseudo $\delta m$ distributions from a CLEO II.V $D^+ \rightarrow \bar{K}^{*0}e^+\nu_e$ signal MC sample are overlaid. The overlay of background $\delta m$ and pseudo $\delta m$ is given in Fig. 3.5 (right). A CLEO II.V background continuum MC sample was used for the background plot. To select background events, all events containing the signal decay mode were discarded using generator-level decay product information. In addition to the analysis cuts, a tight $K\pi$ mass cut, $0.87 < M_{K\pi} < 0.92$ GeV, which corresponds to the full width of the $K^*$, was applied to make these plots. Based on these studies, I found that the anal-
Figure 3.4: Fit of the $\delta m$ peak (left) and that of the pseudo $\delta m$ peak (right) with single Gaussian function. A $\pi^0$ mass sideband subtraction was used in these histograms.

ysis with the $\nu$ reconstruction reduces the statistical error by 15% to 21% compared to that in the pseudo $\delta m$ analysis. These differences were consistent with a study with continuum MC (refer to Table B.1 in APPENDIX B). However, it is important to understand how much peaking one should expect from the background where the signal is expected since the method to choose one of the three solutions for $|\vec{p}_L|$ tends to bias the $\delta m$ distribution of the background. This is described in the section about systematic errors (see section 5.2).

3.4 Fitting Process

3.4.1 Overview

There are two quantities to be fitted to obtain the number of signal events: $M_{K\pi}$ and $\delta m$. To use all of this information, I decided to fit for both of the quantities. Two successive fits were used instead of a two dimensional fit. The process of the fits were:

1. A data sample was divided into 50 bins (25 bins for the muon mode because of its small statistics) over the $\delta m$ range from 0.135 GeV to 0.235 GeV.
Figure 3.5: Overlay of $\delta m$ and pseudo $\delta m$ distribution from a signal MC sample (left) and background (right) events. A $K\pi$ mass cut, $0.87 < M_{K\pi} < 0.92$ GeV, was used for both of the figures.

2. For each $\delta m$ bin, the $K\pi$ mass distribution was plotted and the number of $K^*$’s was extracted with a $K\pi$ mass fit.

3. The extracted number of $K^*$’s were plotted as a function of $\delta m$.

4. The number of signal events were obtained from a fit of this $\delta m$ plot.

3.4.2 $K\pi$ Mass Fit

$K\pi$ mass fits to data are shown in Fig. 3.6, Fig. 3.7, Fig. 3.8 and Fig. 3.9.

In the $K\pi$ mass fit, the signal shape was described by a $p$-wave $K^*$ Breit-Wigner distribution defined by [26]

$$\frac{M_{K^*} \times \Gamma}{(M_{K\pi}^2 - M_{K^*}^2)^2 + M_{K^*}^2 \times \Gamma^2}$$

(3.1)

where

$$\Gamma = \frac{p^3 \times M_{K^*}}{p_f^3 \times M_{K\pi}} \Gamma_{K^*}$$

(3.2)
Figure 3.6: $K\pi$ mass fits in $D^+ \rightarrow K^*0 e^+ \nu_e$ CLEO II.V analysis. From left to right and from top to bottom, $K\pi$ mass fit for the first, the second, the third, the fourth, the twentieth, and the fortieth $\delta m$ bin.
Figure 3.7: $K\pi$ mass fits in $D^+ \rightarrow \bar{K}^*0e^+\nu_e$ CLEO II analysis. From left to right and from top to bottom, $K\pi$ mass fit for the first, the second, the third, the fourth, the twentieth, and the fortieth $\delta m$ bin.
Figure 3.8: $K\pi$ mass fits in $D^+ \to \bar{K}^*\mu^+\nu_\mu$ CLEO II.V analysis. From left to right and from top to bottom, $K\pi$ mass fit for the first, the second, the third, the fourth, the tenth and the twentieth $\delta m$ bin.
Figure 3.9: $K\pi$ mass fits in $D^{+} \rightarrow \bar{K}^*0\mu^{+}\nu_{\mu}$ CLEO II analysis. From left to right and from top to bottom, $K\pi$ mass fit for the first, the second, the third, the fourth, the tenth and the twentieth $\delta m$ bin.
Figure 3.10: Fit of QQ tagged signal $M_{K\pi}$ with the $p$-wave $K^*$ Breit-Wigner distribution.

and $p$ is the momentum of the $K$ in the $K^*$ rest frame and $p_r$ is the same quantity for $M_{K\pi} = M_{K^*}$. A $p$-wave Breit-Wigner distribution was used because in the decay $K^{*0} \rightarrow K^-\pi^+$, a vector particle decays to two pseudo scalar particles. To conserve angular momentum, the orbital angular momentum should be one and the spatial function between $K^-$ and $\pi^+$ is $p$-wave. The effective values of $\Gamma_{K^*}$ and the $M_{K^*}$ were obtained using signal MC to take account of the smearing. The fit to get the signal shape of the $K\pi$ mass fit is shown in Fig. 3.10. QQ tagged MC signal events in which a $p$-wave $K^*$ Breit-Wigner distribution was used for generating the $K^*$ mass distribution were fitted with the $p$-wave $K^*$ Breit-Wigner distribution. The measured $K^*$ mass was found to be the same as the true $K^*$ mass, 0.896 GeV, and the $K^*$ full width was slightly larger than 0.0505 GeV, the nominal full width of the $K^*$. [11]

An analytic threshold function

\[ (M_{K\pi} - 0.63)^\alpha \times e^{\beta(M_{K\pi} - 0.63) + \gamma(M_{K\pi} - 0.63)^2} \]  

(3.3)

was used to describe the shape of the background. The parameters $\alpha$, $\beta$ and $\gamma$ in this background function were floated in the fits.
Figure 3.11: $\delta m$ fits in CLEO II.V (left) and CLEO II (right) $D^+ \rightarrow K^{*0} e^+ \nu_e$ analyses.

The window of a $K\pi$ mass plot, from 0.64 GeV to 1.24 GeV, was selected wide enough so that the background shape can be determined by the fits. The bin width of the $K\pi$ mass plot was chosen to be 20.0 MeV which is about half of the $K^*$ width.

### 3.4.3 $\delta m$ Fit

The $\delta m$ fits of the CLEO II.V and the CLEO II electron and muon mode data are shown in Fig. 3.11 and Fig. 3.12, respectively.

The signal shapes for the $\delta m$ fits were obtained from signal MC samples. 52673 and 63905 $D^+ \rightarrow K^{*0} e^+ \nu_e$ signal MC events were generated with the ISGW2 model to obtain the signal shapes of the electron mode for CLEO II and CLEO II.V data samples, respectively. For the muon mode, 52000 and 80000 $D^+ \rightarrow K^{*0} \mu^+ \nu_\mu$ signal MC events generated with the ISGW2 model were used to extract the $\delta m$ signal shapes for CLEO II and CLEO II.V data. With these signal MC samples, $\delta m$ histograms were made via the $K\pi$ mass fits for each $\delta m$ bin. These signal shapes were produced separately for the electron and the muon mode and for each of CLEO II and CLEO II.V data, and were used as the signal fit functions in the $\delta m$ fits for data. All of these signal MC samples were independent of the signal MC samples used in
Figure 3.12: $\delta m$ fits in CLEO II.V (left) and CLEO II (right) $D^+ \rightarrow \bar{K}^*\eta_\mu^+\nu_\mu$ analyses.

the kinematic cut optimization or MC samples for the efficiency calculations.

The background shape in the $\delta m$ fit was described by a continuous function

$$
\begin{align*}
A_{bkg} \left( (\delta m - 0.135)^Q_{st} + \frac{A_r}{\sigma} e^{-\frac{(\delta m - 0.141)^2}{2\sigma^2}} \right) & \quad ; \quad 0.135 \leq \delta m < 0.141 \text{GeV}, \\
A_{bkg} \left( (\delta m - 0.135)^Q_{st} e^{-B_{st}(\delta m - 0.141)} + \frac{A_r}{\sigma} e^{-\frac{(\delta m - 0.141)^2}{2\sigma^2}} \right) & \quad ; \quad 0.141 \leq \delta m < 0.235 \text{GeV}.
\end{align*}
$$

This $\delta m$ background function was designed to accommodate the characteristic that the neutrino momentum selection method makes an excess at $M_{D^+} - M_{D^+} = 0.141$ GeV, not only for signal but also for background. The values of the parameters were determined using background MC. To consider only background events, all events containing the decay mode, $D^+ \rightarrow \bar{K}^*\ell^+\nu_\ell$, were discarded with QQ tagging. Then, among the $K^*$ candidates, only real $K^*$’s were selected by using QQ tagging. Since the $K\pi$ mass fit works well in extracting the number of real $K^*$’s, obtaining real $K^*$’s with QQ tagging would be equivalent to executing $K\pi$ mass fits. After plotting the number of real $K^*$’s as a function of $\delta m$, the $\delta m$ fit was processed with the above $\delta m$ background function with all variables floating. Once the values of $Q_{st}$, $B_{st}$, $A_r$ and $\sigma$ were obtained from this fit, these values were fixed in the $\delta m$ fit for data to measure the branching fraction. The values of these parameters used for the fit for
the electron mode were $Q_{sl} = 0.85$, $B_{sl} = 6.79$, $A_r = 6.44 \cdot 10^{-6}$, and $\sigma = 1.48 \cdot 10^{-3}$. For the muon mode, $Q_{sl} = 0.91$, $B_{sl} = 3.20$, $A_r = 1.91 \cdot 10^{-5}$, and $\sigma = 5.70 \cdot 10^{-3}$ were used. Because the values of these parameters were consistent between CLEO II.V and CLEO II, the same values were used in the $\delta m$ fits for both data sets. The number of reconstructed events obtained using these fits is described in section 5.1.

Validity of using MC to determine the background shape was checked by asking the following questions:

- Is the expression to describe the background function appropriate?
- Is it enough to estimate the $\delta m$ background shape with only real $K^*\pi$ in background events?

To quantify appropriateness of using the background function in equation 3.4, these background $\delta m$ histograms were used as the $\delta m$ background fit functions and the results were compared with the result obtained using the $\delta m$ background function in equation 3.4. The variation of the branching fraction measurement resulting from changing $\delta m$ background fit functions is described in the section of the systematic error estimation (see section 5.2).

To address the latter question, a different background MC sample was used. After using QQ tagging to reject signal events, a $\delta m$ background histogram was made using all events whose $M_{K^\pi}$ satisfy $0.87 < M_{K^\pi} < 0.92$ GeV, the full width of $K^*$. The latter sample contains not only real $K^*$'s but also random combinations and $\rho^0$ feed-through events. The shape of this $\delta m$ background histogram was compared with that of the original $\delta m$ background histogram. As can be seen in Fig. 3.13, these two histograms agree bin by bin when they are normalized to equal areas. For a more quantitative comparison, the two $\delta m$ background histograms were used as $\delta m$ background fit functions in the $\delta m$ fits for CLEO II.V data and the yields were compared. When the $\delta m$ background histogram obtained using the QQ tagging method was used as $\delta m$ background fit function, the number of electronic mode events in the CLEO II.V data was 3922.8 $\pm$ 292.4. The electronic mode yield was 3901.8 $\pm$ 294.4 when the $\delta m$ background histogram made by the $M_{K^\pi}$ mass cut was used. Fig. 3.14 shows these $\delta m$ fits for the CLEO II.V data. Furthermore, these yields were consistent with that in
Figure 3.13: Background \( \delta m \) shape comparison. The histogram of the square data points was made with the QQ tagged real \( K^* \)'s and the triangle data point distribution was obtained using the \( K\pi \) mass cut, \( 0.87 < M_{K\pi} < 0.92 \) GeV.

\[
\begin{array}{c}
\text{Data Points} \\
\text{Fit} \\
\text{\ldots Background} \\
N &= 3922.8 \pm 292.4 \\
\chi^2 &= 36.879 / (50-2) \\
\text{C.L.} &= 87.846 \% \\
\end{array}
\]

Figure 3.14: \( D^+ \rightarrow K^{*0} e^+ \nu_e \) mode \( \delta m \) fits for the CLEO II.V data using the \( \delta m \) background histograms using the QQ tagging method (left) and the \( K\pi \) mass cut, \( 0.87 < M_{K\pi} < 0.92 \) GeV (right).
the standard analysis method, 3899.9 ± 295.4 (refer to section 5.1). This consistency implies that the consideration of real $K^*$'s for the $\delta m$ background shape estimation is enough.

### 3.5 Particle Identification

It has been demonstrated that using particle identification will introduce about a 2% systematic error per track. [25] Therefore it is necessary to study how much the particle identification helps to reduce the overall uncertainty before I decide whether or not I should use the particle identification in this analysis. If the reduction of the statistical error is less than or comparable to the increment of the systematic error due to using particle identification, it would be better not to use it. For this study, the following issues were studied using a continuum MC sample:

- Whether both $\pi$ and $K$ candidates should be identified or if identifying one of them is more optimum if the particle identification were to be used. If identifying one particle is better, then which particle to identify should be identified.

- How much does $dE/dx$ help to reduce the overall uncertainty?

First, since my ability to distinguish $\pi$ and $K$ is limited, I looked at the momentum range of the $\pi$ and $K$ candidates using a background MC sample. They are shown in Fig. 3.15. Fig. 3.15 shows that momenta of about 50% of $\pi$ and $K$ candidates are greater than 0.8 GeV. For these high momentum particles, $dE/dx$ would not give a good separation between $\pi$ and $K$.

Fig. 3.15 also shows that the momenta of $\pi$ and $K$ candidates have similar distributions. As a result, $K$ candidates would have many more fakes since there are many more $\pi$'s than $K$'s in my data sample. Fig. 3.16 is a two dimensional scattered plot of $\pi$ and $K$ candidate momenta, which shows some level of correlation between the two. When the $\pi$ candidate momentum is high, then the $K$ candidate momentum tends to be low and vice versa. However this correlation turns out to be not strong enough to help reduce background substantially when only the lower-momentum particle is identified as shown below.
Figure 3.15: Momentum distributions of π candidates (left) and K candidates (right).

Using $dE/dx$ information, the probability for a kaon to be misidentified as a pion is comparable with that for a pion to be misidentified as a kaon. Since there are many more pions than kaons at CLEO, the ratio of the number of true pions to the number of kaons misidentified as pions is greater than the opposite case. Therefore, if particle identification is used for only one track, it should be used for the $K$ candidate. This can also be confirmed in Fig. 3.17. Fig. 3.17 shows $M_{K\pi}$ plots of a generic continuum CLEO II.V MC sample for three cases: (i) without using $dE/dx$, (ii) with SGKADI\textsuperscript{5} $< 3.0$, and (iii) with SGKADI $< 2.0$ in addition to the analysis cuts in Table 3.1 in the signal $\delta m$ region, $0.137 \leq \delta m \leq 0.149$ GeV. Each of $M_{K\pi}$ plots in Fig. 3.17 also shows relative portions of four components: (i) real $\pi$ and real $K$ combinations (open part), (ii) real $\pi$ and fake $K$ combinations, (iii) fake $\pi$ and real $K$ combinations, and (iv) fake $\pi$ and fake $K$ combinations. To obtain the real particle identification information, the QQ information of tracks was used. Fig. 3.17 reveals that fake $K$’s are the dominant background source in $M_{K\pi}$ plots. When real $\pi$ and real $K$ combinations are regarded as signal ($S$), which are what I want to choose using $dE/dx$, and other cases are taken

\textsuperscript{5} A variable indicating how many $\sigma$’s away the likelihood of a particle to be a kaon is according to its $dE/dx$ information.
Figure 3.16: Two-dimensional plot of π candidates and K candidates momenta.

as background (N), the effectiveness of dE/dx was tested by comparing S²/(S + N) for three cases: (i) without using dE/dx, (ii) with SGKADI < 3.0, and (iii) with SGKADI < 2.0 in Fig. 3.17. The values of S²/(S + N) in the three cases of Fig. 3.17 were (i) 610.7 ± 20.8, (ii) 624.6 ± 20.5, and (iii) 608.9 ± 20.3, respectively. Therefore, within the errors all three cases gave the same S²/(S + N)’s.

As another study, S²/N values obtained (i) without using dE/dx, (ii) with SGKADI < 3.0, and (iii) with SGKADI < 2.0 were compared, where S and N respectively indicate the areas of M_{Kπ} distributions made with signal MC events and generic continuum MC background events satisfying the analysis cuts and 0.137 ≤ δm ≤ 0.149 GeV. The obtained S²/N values were: (i) 1326.3 ± 80.4, (ii) 1363.0 ± 82.8, and (iii) 1328.5 ± 82.0 for these three cases, respectively.

The results of these studies show that using particle identification would not give much gain in reducing statistical errors. These results were also confirmed by the comparison of analysis results obtained with and without dE/dx. This comparison revealed that dE/dx does not reduce the statistical errors in this analysis. Consequently I did not use any particle identification for this work.
Figure 3.17: Mass of $\pi K$ with fake contributions when $dE/dx$ information was not used (left), when SGKADI < 3.0 was used (middle), and when SGKADI < 2.0 was used (right).

### 3.6 $D^+$ Lifetime Cut

The major difference between CLEO II and CLEO ILV data samples is in the presence of the SVX for the latter. The CLEO II-V SVX in principle allows precise secondary vertexing. Using this precise secondary vertexing, many analyses at CLEO, like measurements of the $D$ meson lifetime [27] and $D^0 - \bar{D}^0$ mixing, [28] have been successful. Since $D^+$ meson has a long lifetime, this property may help reducing background level. In the kinematic cut optimization process of this analysis, the $D$ meson lifetime was considered as one of the kinematic cut candidates. In the final kinematic cut list, however, the $D^+$ meson lifetime was excluded since it did not improve $S^2/N$ once the $|\vec{p}_\pi|$ cut and the $|\vec{p}_{K^{\ast\ell}}|$ cut were made.
CHAPTER 4
NORMALIZATION MODE ANALYSIS

The decay chain $D^{*+} \rightarrow D^+ \pi^0$ and $D^+ \rightarrow K^- \pi^+ \pi^+$ was used for the normalization mode. One of the advantages in using this mode is that many of the systematic errors of this analysis are canceled out. Furthermore, the branching ratio of $D^+ \rightarrow K^- \pi^+ \pi^+$ is well measured: $9.0 \pm 0.6\%$. [11]

4.1 Selection of Normalization Mode Events

The same global, track, and $\pi^0$ selection cuts used in the $D^+ \rightarrow \bar{K}^* l^+ \nu_l$ analysis were also applied to this normalization mode. Most of the kinematic cuts were also the same but the cut on $|\vec{p}_D|$ was tuned so that the cut on $|\vec{p}_D|$ of the normalization mode matches that for $D^+ \rightarrow \bar{K}^* l^+ \nu_l$. As a result, the effect of uncertainty in the fragmentation of $D^+$ is minimized. Fig. 4.1 shows the $D$ momentum distributions in $D^+ \rightarrow \bar{K}^* l^+ \nu_l$ and $D^+ \rightarrow K^- \pi^+ \pi^+$ in signal MC events which pass the $|\vec{p}_{K\pi\pi}|$ and $|\vec{p}_{K\pi\pi}|$ cuts as well as other cuts including having $\delta m$ in the range from 0.137 to 0.149 GeV.

Unlike $D^+ \rightarrow K^- \pi^+ \pi^+$, one of the final particles, the $\nu$ in $D^+ \rightarrow \bar{K}^* l^+ \nu_l$, is not detected. As a result, the reconstructed $|\vec{p}_D|$ in $D^+ \rightarrow \bar{K}^* l^+ \nu_l$ is smeared more than that in $D^+ \rightarrow K^- \pi^+ \pi^+$. Since the charm fragmentation is more relevant to the generated $|\vec{p}_D|$ than the reconstructed, generated $|\vec{p}_D|$’s were used for tuning the $|\vec{p}_D|$ cut.

Table 4.1 shows the kinematic cuts for this normalization mode.

4.2 Analysis Methods

To obtain the number of events in data and the efficiency of the normalization mode using signal MC events, two successive fits of $D^+$ mass and $\delta m$ were used, an approach
Figure 4.1: Comparison of $D$ momentum distributions between $D^+ \rightarrow \bar{K}^* l^+ \nu_l$ and $D^+ \rightarrow K^- \pi^+ \pi^+$. The generator-level (left) and the reconstructed (right) $D$ momentum distributions are shown.

<table>
<thead>
<tr>
<th>variable</th>
<th>value</th>
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<tbody>
<tr>
<td>$</td>
<td>\vec{p}_{\pi^+}</td>
</tr>
<tr>
<td>$</td>
<td>\vec{p}_{K^-}</td>
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<tr>
<td>$</td>
<td>\vec{p}_{\pi^0}</td>
</tr>
<tr>
<td>$</td>
<td>\vec{p}_{D^+}</td>
</tr>
<tr>
<td>$</td>
<td>M_{\gamma\gamma} - M_{\pi^0}</td>
</tr>
</tbody>
</table>
similar to that used in the semileptonic analysis.

The data were divided into fifty $\delta m$ bins from $0.135$ GeV to $0.185$ GeV. Since the $\delta m$ peak of $D^+ \rightarrow K^-\pi^+\pi^+$ is narrower than that of $D^+ \rightarrow \bar{K}^{*0} l^+ \nu_l$, the bin width of the normalization mode $\delta m$ histogram, 1.0 MeV, is finer than the 2.0 MeV bin width of $D^+ \rightarrow \bar{K}^{*0} l^+ \nu_l$ $\delta m$ histogram. For each of the $\delta m$ bins, a $D^+$ mass fit was executed to extract the number of $D^+$'s in that $\delta m$ bin. In these $D^+$ mass fits, two Gaussian functions were used to represent the signal shape and a second order Chebyshev polynomial function was used for the background fit. The means of the two Gaussian functions were required to be the same. The parameters determining the shape of the signal were determined by fitting the $D^+$ mass distribution of QQ tagged signal MC events. The $D^+$ mass was allowed a small variation since the mass scale of CLEO MC is not certain to a few MeV. The averages of the $D^+$ masses over $\delta m$ bins were $1.8694 \pm 0.0018$ GeV and $1.8699 \pm 0.0011$ GeV for CLEO II $D^+ \rightarrow K^-\pi^+\pi^+$ signal MC and CLEO II data, respectively. For CLEO II.V signal MC and data, the averages of the $D^+$ masses over $\delta m$ bins were $1.8693 \pm 0.0011$ GeV and $1.8698 \pm 0.0010$ GeV. The difference is consistent within the systematic error on the mass. Fig. 4.2 shows the $D^+$ mass fits with two Gaussian functions for CLEO II.V and CLEO II QQ tagged normalization mode MC events. The parameters of the second order Chebyshev polynomial function, representing the background in the $D^+$ mass fits for data, were floated.

The extracted numbers of $D^+$'s were plotted as a function of $\delta m$. Then, this $\delta m$ plot was fitted to obtain the number of the normalization mode events. In this $\delta m$ fit, the signal $\delta m$ shape was made with the QQ tagged normalization mode MC events. Since QQ tagged $D^+$'s are already real $D^+$'s, the signal shape of the $\delta m$ fit was determined without $D^+$ mass fits. The background in the $\delta m$ fit was described with an analytic function,

$$A_{bkg} \times (\delta m - 0.135)^{Q_{had}} e^{-B_{had}(\delta m - 0.135)}.$$  \hspace{1cm} (4.1)

The parameters $Q_{had}$ and $B_{had}$ in equation 4.1 were floated in the $\delta m$ fit. The number of reconstructed events in the $D^+ \rightarrow K^-\pi^+\pi^+$ analysis is given in section 5.1.
Figure 4.2: $D^+$ mass fits with two Gaussian functions for CLEO II.V (left) and CLEO II (right) QQ tagged normalization mode MC events.

Since the dominant background source in the $\delta m$ distribution after $D^+$ mass fits is the combination of real $D^+$’s and fake $\pi^0$’s, the appropriateness of this $\delta m$ background function was tested with $\pi^0$ mass sideband data. The region of $3.5\sigma \leq |M_{\gamma\gamma} - M_{\pi^0}| \leq 7.5\sigma$ was chosen as the $\pi^0$ mass sideband. The $\delta m$ plot for CLEO II.V $\pi^0$ mass sideband band data obtained with $D^+$ mass fits was fitted with the $\delta m$ background fit function. In this $\delta m$ background fit, two points were checked. One was whether the fit itself was reasonable. The other was whether or not the values of the parameters $Q_{\text{had}}$ and $B_{\text{had}}$ in equation 4.1 were consistent with those in the normal $\delta m$ fit for the $\pi^0$ signal band data. The $\delta m$ background fit gave reasonable fit results as shown in Fig. 4.3. Furthermore, the values of $Q_{\text{had}}$ and $B_{\text{had}}$ in equation 4.1 in the $\delta m$ background fit were very consistent with those in the normal $\delta m$ fit (refer to Fig. 4.4). These two results confirm that the $\delta m$ background in equation 4.1 is an appropriate representation of the background in the $\delta m$ distribution after the $D^+$ mass fits. The results of the $\delta m$ fits for CLEO II.V and CLEO II data are given in Fig. 4.4.
Figure 4.3: A $\delta m$ background fit with the $\delta m$ background function in equation 4.1 for CLEO II.V $\pi^0$ mass sideband data.

Figure 4.4: $\delta m$ fits in CLEO II.V (left) and CLEO II (right) $D^+ \rightarrow K^- \pi^+ \pi^+$ analyses. These $\delta m$ plots were made with $D^+$ mass fits.
CHAPTER 5
RESULTS OF MEASUREMENTS AND SYSTEMATIC ERRORS

5.1 Results of Fits

The branching fraction measurements of $D^+ \to \bar{K}^{*0} l^+ \nu_l$ with respect to $D^+ \to K^- \pi^+ \pi^+$ were separately done for the $e$ and $\mu$ modes. Furthermore, for each lepton mode, the branching fraction measurements were separately executed for CLEO II and CLEO II.V data sets.

For the branching fraction measurement of the $e$ mode with the CLEO II.V data sets, a total of 127905 $D^+ \to \bar{K}^{*0} e^+ \nu_e$ signal MC events were generated with the ISGW2 model. Among them, 63905 events were used to obtain the signal shape in the $\delta m$ histogram and the rest were used for the efficiency calculation. For CLEO II, 102775 signal MC events were produced and 52673 of them were used to extract the $\delta m$ signal shape and the rest were used to find the efficiency. The efficiencies for $D^+ \to \bar{K}^{*0} e^+ \nu_e$ were found to be 5.45% and 5.55% for CLEO II.V and CLEO II respectively. The number of reconstructed events for the CLEO II.V data is $3899.9 \pm 295.4$ and $2330.8 \pm 193.7$ for the CLEO II data as described in chapter 3.

In the $\mu$ mode, of 177396 $D^+ \to \bar{K}^{*0} \mu^+ \nu_\mu$ ISGW2-model signal MC events, 80000 events were used to extract the $\delta m$ signal shape and the rest were used for the efficiency calculation. For the CLEO II $\mu$ mode, a total of 52000 ISGW2-model signal MC events were used for the $\delta m$ signal shape estimation and 50775 events were used for the efficiency calculation. The efficiencies were 0.085% and 0.096% for the CLEO II.V and CLEO II $\mu$ modes, respectively. The number of reconstructed events in the CLEO II.V and CLEO II $\mu$ modes were $630.4 \pm 93.3$ and $443.4 \pm 75.0$.

The results of the $\delta m$ fits for the muon mode CLEO II.V and CLEO II data are shown in Fig. 3.12.
For the normalization mode, a total of 49354 $D^+ \to K^-\pi^+\pi^+$ MC events were used to obtain the efficiency for CLEO II.V. In the CLEO II normalization mode analysis, the efficiency was calculated from 53409 MC events. With the $D^+$ mass and $\delta m$ fit method, the calculated efficiencies of the normalization mode were 6.34% and 6.34% in the CLEO II.V and CLEO II analyses. The numbers of detected events in the normalization modes for CLEO II.V and CLEO II were $10018.0 \pm 158.7$ and $5169.1 \pm 119.6$, respectively.

5.2 Systematic Error Estimation

Before the final results are presented, I must estimate the systematic errors. The following items have been studied as potential sources of systematic errors in this analysis:

- fits,
  - $\delta m$ signal shape estimation with signal MC,
  - $\delta m$ background shape estimation method with continuum MC,
  - data and continuum MC difference,
- lepton identification in data,
- fake lepton contribution in data,
- model dependence of signal MC,
- efficiency variation due to fragmentation difference between data and MC,
- feed down from semileptonic decays of higher $K^*$ resonances from $D^+$,
- contribution from $D^+ \to K^*0\pi^0l^+\nu_l$ mode,
- contribution from non-resonant $D^+ \to K^-\pi^+l^+\nu_l$ decay mode,
- uncertainty in the normalization mode analysis, and
- track finding efficiency in the normalization mode.
5.2.1 Fits

In the semileptonic analysis, two types of fits were used: the $K\pi$ mass fit and the $\delta m$ fit. The systematic error related to the $K\pi$ mass fit was neglected. This is partially because:

- All parameters of the background function for the $K\pi$ mass fit were floated. As a result, uncertainties in the background shape due to parameter variations are included in the statistical error.

- The mass and width of the $K^*$ extracted from signal MC gave good fit results for data as can be seen in the $K\pi$ mass fit plots in section 3.4. Because the width of the reconstructed $K\pi$ mass distribution is mostly determined by $\Gamma_{K^*}$, not by detector resolution, and the variation of $M_{K^*}$ due to the detector resolution (0.0008 GeV in Fig. 3.10) is much less than $\Gamma_{K^*}$, the signal shape of the $K\pi$ mass fit does not affect the results. Furthermore, the numerical measures show that the fit quality was good overall and the $\chi^2/d.o.f.$ in the signal region is also good (20.9/(30 – 5) and 17.5/(30 – 5) for the $K\pi$ mass fit for the electron mode CLEO II.V and CLEO II data, respectively. For the muon mode, 21.1/(30 – 5) and 26.1/(30 – 5) for CLEO II.V and CLEO II, respectively.).

However, the last, and most important question is whether the $K\pi$ mass fit background function in the equation 3.3 is an appropriate description of the background shape in the fit. To address this question, I studied how well the $M_{K\pi}$ background fit function describes the shapes of the $M_{K\pi}$ histograms from continuum MC background events. I fitted these histograms with the background function. Using the QQ tagging technique, all events containing the signal decay sequence were excluded. The $\delta m$ window from 0.135 to 0.235 GeV was divided into 25 bins. For each of $\delta m$ bins, a $M_{K\pi}$ fit with the background function in the equation 3.3 was executed with all parameters floating. The fit results for the first, fifth, tenth, fifteenth, twentieth, and twenty-fifth $\delta m$ bin are given in Fig. 5.1. All of the $M_{K\pi}$ fits gave reasonable fit results. To be more quantitative, $M_{K\pi}$ fits artificially including a signal fit function in equation 3.1 in addition to the background function (equation 3.3) were executed
for the continuum MC background events. The number of $K^*$'s extracted from these
fits are plotted as a function of $\delta m$ in Fig. 5.2. It shows that the "signal" yields are
consistent with zero and furthermore does not have a $\delta m$ peak where the real signal
events would. Based on these observations, the appropriateness of the $K\pi$ mass fit
background function was not considered as a systematic error source.

In the $\delta m$ fit, three systematic error sources were studied. The first source is the
signal shape estimation. As explained in section 3.4, signal MC samples were used to
extract the signal shape. Because of the finiteness of the signal MC sample size, there
are errors in the signal shape estimation. When a histogram is used as a fit function
in Mn_Fit, [29] there are two options. One is to set the errors of the fit function
histogram to be zeros and the other is to use the associated errors of the histogram
in the fit. All of the results in section 5.1 were obtained using the first option since
the finiteness of the signal MC sample sizes was taken into account by including the
errors in the $\delta m$ signal shape histogram using the second option. The normalization
parameter of background was allowed to float. The quadratic difference of the results
of these two fit options was used as the systematic error related to the determination
of the $\delta m$ signal shape. The estimated systematic errors were 3.9%, 4.3%, 6.3%,
and 5.8% for the electron mode CLEO II V and CLEO II data and the muon mode
CLEO II V and CLEO II data, respectively.

To quantify the systematic error in the background $\delta m$ shape estimation, four
different background $\delta m$ shapes were used in the $\delta m$ fit and the root-mean-square (RMS) of the four fit results was used as the systematic error. The four estimates of
the $\delta m$ background shapes were:

- The $\delta m$ background function in equation 3.4. As explained in subsection 3.4.3,
the parameters $Q_{sl}$, $B_{sl}$, $A_{r}$ and $\sigma$ were obtained from the fit of QQ tagged $K^*$'s
with this background function for continuum MC background events.

- The function in equation 3.4 as above. However, its parameters $Q_{sl}$ and $B_{sl}$ were
floated in the electron mode. In the muon mode, the background normalization
parameter had a negative value when both $Q_{sl}$ and $B_{sl}$ parameters were floated.
Based on the assumption that the systematic error associated with the $\delta m$
Figure 5.1: $M_{K\pi}$ fits with $M_{K\pi}$ background fit function for the background $M_{K\pi}$ histograms for the first (top left), fifth (top right), tenth (middle left), fifteenth (middle right), twentieth (bottom left), and twenty-fifth (bottom right) $\delta m$ bin.
background shape in the muon mode is same as that in the electron mode, the electron mode fit results were also used for the muon mode. The parameters $A_r$ and $\sigma$ were fixed to the values of the normal analysis in the fits.

- A $\delta m$ histogram of background continuum MC events having real $K^*$’s (QQ tagged).

- A $\delta m$ distribution of background continuum MC events satisfying $0.87 < M_{K\pi} < 0.92$ GeV.

Table 5.1 shows the numbers of detected events with the four different $\delta m$ background shape estimations for the CLEO II.V and CLEO II data and their normalized RMS’s. These normalized RMS’s were assigned as the corresponding systematic errors.

As the last systematic error source in the fit studied was the difference between data and MC. If data and MC have different shapes of $M_{K^*}$ or $\delta m$ in signal or in background, then the results of this analysis would be biased since the parameters of the background functions or histograms were either obtained from or tested using MC samples. The systematic error from this source was estimated using a $\pi^0$ mass sideband, $3.5\sigma < |M_{\gamma\gamma} - M_{\pi^0}| < 15.0\sigma$. The $\pi^0$ mass sideband data were analyzed as
Table 5.1: The numbers of detected events with the four different \( \delta m \) background shape estimations in \( D^+ \to \bar{K}^{*0} l^+ \nu_l \) data.

<table>
<thead>
<tr>
<th></th>
<th>CLEO II.V</th>
<th>CLEO II</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta m ) bgr. function in equation 3.4</td>
<td>( D^+ \to \bar{K}^{*0} e^+ \nu_e ) ( \Delta^2 ) ( \bar{K}^{*0} \mu^+ \nu_\mu )</td>
<td>( D^+ \to \bar{K}^{*0} e^+ \nu_e ) ( \Delta^2 ) ( \bar{K}^{*0} \mu^+ \nu_\mu )</td>
</tr>
<tr>
<td>( \delta m ) bgr. function in equation 3.4 with floating ( Q_{K'} ) and ( B_{st} )</td>
<td>( 3723.8 \pm 283.2 ) ( 627.5 \pm 116.5 )</td>
<td>( 2315.7 \pm 193.7 ) ( 353.5 \pm 84.7 )</td>
</tr>
<tr>
<td>( \delta m ) bgr. hist. with QQ tagged ( K^{*-} )s</td>
<td>( 3569.0 \pm 690.7 )</td>
<td>( 2199.2 \pm 283.9 )</td>
</tr>
<tr>
<td>( \delta m ) bgr. hist. with the ( K_{\pi} ) mass cut</td>
<td>( 3745.7 \pm 280.4 ) ( 612.2 \pm 121.6 )</td>
<td>( 2298.3 \pm 193.3 ) ( 349.4 \pm 88.4 )</td>
</tr>
<tr>
<td>normalized RMS</td>
<td>( 1.92% ) ( 2.78% )</td>
<td>( 3.06% ) ( 1.02% )</td>
</tr>
</tbody>
</table>

\( \pi^0 \) signal band data and the \( \delta m \) histogram was made with the \( M_{K\pi} \) fits. The \( \pi^0 \) mass sideband events in a continuum MC sample were also analyzed with the same method. Then, the \( \delta m \) histogram from data was fitted with the \( \delta m \) histogram obtained with continuum MC \( \pi^0 \) mass sideband events and the “signal” \( \delta m \) histogram, which was used in the normal analysis \( \delta m \) fits. Because the \( \pi^0 \) mass sideband events do not include signal decay chains, the number of “signal” events in this \( \delta m \) fit is expected to be zero if the continuum MC simulated the background in the data well. This number of signal events obtained by the \( \delta m \) fit is a useful measure of the difference between data and continuum MC. The process was:

1. All analysis cuts except the \( \pi^0 \) mass cut were applied. Then, the number of entries in the \( \pi^0 \) mass sideband was compared with that in the \( \pi^0 \) mass signal band. The ratio of these numbers of entries were used for the normalization of the \( \pi^0 \) mass sideband data to the data in the \( \pi^0 \) mass signal band.

2. The number of “signal” events and statistical errors using the \( \pi^0 \) mass sideband data were obtained with the \( \delta m \) fit explained above. The sum of the central value and the one \( \sigma \) statistical error, “one \( \sigma \) upper limit”, was normalized with the ratio obtained in the first step above.

3. The normalized “one \( \sigma \) upper limit” in the second step was divided by the
Figure 5.3: $\delta m$ fits of the electron mode CLEO II.V (left) and CLEO II (right) $\pi^0$ mass sideband data with the signal $\delta m$ shapes and $\delta m$ histograms obtained from continuum MC $\pi^0$ mass sideband events.

number of real signal events obtained from the signal-band data and this was considered to be the systematic error from the difference between data and continuum MC.

This process was executed separately for the CLEO II.V and the CLEO II electron mode. The ratios of the number of entries in the signal and the $\pi^0$ mass sideband were 0.30 for both CLEO II.V and CLEO II. The unnormalized number of signal events obtained in the above second step was $3.0 \pm 599.0$ and $20.9 \pm 385.9$ for CLEO II.V and CLEO II, respectively. Therefore, the “one $\sigma$ upper limits” for CLEO II.V and CLEO II are 602.0 and 406.8, respectively. After multiplying these “one $\sigma$ upper limits” with the ratios of the number of entries in the signal and the $\pi^0$ mass sideband (0.3), I obtained the systematic errors for the electron mode of 0.045 and 0.053 for CLEO II.V and CLEO II, respectively. The $\delta m$ fits to the $\pi^0$ sideband data for the CLEO II.V and CLEO II electron modes are given in Fig. 5.3.

Because of small statistics, this systematic error for the muon mode was not estimated. The systematic errors for the muon mode were assumed to be the same as those of the electron mode.
5.2.2 Lepton Identification Efficiency

The systematic error of the electron identification efficiency was estimated based on a radiative-bhabha embedding study. As in the CLEO standard electron efficiency studies for $B\bar{B}$ events, radiative bhabha events were embedded into hadron events and the electron identification efficiencies were measured. Since this $D^+ \rightarrow \bar{K}^* e^+\nu_e$ analyses was done with continuum events, the characteristics of the event shapes of continuum events must be taken into account. In my study, the radiative-bhabha embedded hadron events were made to have an event shape similar to that of $D^+ \rightarrow \bar{K}^* e^+\nu_e$ signal MC events. The event shape of an event was parameterized by two parameters: the thrust and the cosine of the angle between the electron momentum and the thrust axis of the event. The weighting factors were determined from the two dimensional distribution of a $D^+ \rightarrow \bar{K}^* e^+\nu_e$ signal MC sample after applying the analysis cuts in Table 3.1.

The systematic error of the electron identification efficiency was estimated for three sources: (i) quality of embedded radiative bhabha events, (ii) the difference in track and shower matching information of electrons in radiative bhabha events (before embedding) and hadron events (after embedding), and (iii) systematic error in the event shape weighting factors. The estimated systematic errors from the electron identification efficiency are 2.0% for CLEO II.V and 2.0% for CLEO II. Details are described in APPENDIX D.

For the estimation of the systematic error in the muon identification efficiency, CLEO II and CLEO II.V muon identification efficiency studies [31] were used. According to these studies, MC events simulate muon identification very well and the disagreement between data and MC is 0.5%. This 0.5% difference was used as the systematic error in muon identification efficiency for both CLEO II and CLEO II.V.

5.2.3 Fake Lepton Contribution in Data

The lepton fake rate in data depends on the momentum as well as on the polar angle of the lepton candidates. Based on the CLEO II fake electron studies, [33] the electron fake rate is less than 0.3% in the polar angle and momentum range of the electron
candidates in this analysis. From CLEO II and CLEO II.V fake muon studies, [33] the fake rate in this analysis is less than 3.0%.

To quantify the fake lepton contribution in data samples, the analysis was repeated for the data samples with anti-lepton identification cuts. In other words, all tracks identified as leptons with the lepton identification cuts were discarded, and all remaining tracks were used as the potential lepton candidates. However, all of the parameters of the shapes in the fits were kept to be the same as those in normal analysis except parameters \( Q_{sl} \) and \( B_{sl} \) of equation 3.4. \( Q_{sl} \) and \( B_{sl} \) were allowed to float because the \( \delta m \) background shape in the normal analysis dominated by real leptons and the hadron data analyzed here may be different. With these anti-electron candidates, the measured numbers of reconstructed events for the electron mode were 710.27 ± 837.75 and 343.67 ± 542.638 for CLEO II.V and CLEO II data, respectively. Within one \( \sigma \), the upper limits of these measured events are 1548.0 and 886.3. Multiplying these upper limits by the electron fake rate (0.3%) gives 0.12% and 0.11% systematic errors for the CLEO II.V and CLEO II electron mode. For the muon mode, the measured numbers of events from CLEO II.V and CLEO II fake muons were 69.635 ± 285.90 and 41.217 ± 191.82. Then, one \( \sigma \) upper limits of these numbers of events are 355.5 and 233.0 for CLEO II.V and CLEO II, respectively. Multiplying these upper limits by the muon fake rate ( < 3.0%) gives systematic errors for the CLEO II.V and CLEO II of 1.7% and 2.0% due to uncertainties in the rate of fake muons. Fig. 5.4 and Fig. 5.5 show the \( \delta m \) fits for CLEO II.V and CLEO II data in this fake lepton contribution study.

### 5.2.4 Model Dependence of Signal MC

As mentioned, the signal MC samples for this analysis were generated based on the ISGW2 model. To quantify how much the results of this analysis depend on the model of the signal MC samples, five samples of signal MC for \( D^+ \rightarrow \bar{K}^*0 e^+\nu_e \) were generated using HQET-based parameterization where the form factor ratios, \( r_V \) and \( r_2 \) were varied. The ratios \( r_V \) and \( r_2 \) were most precisely measured at E791 [37] to be \( r_V = 1.84 \pm 0.136 \) and \( r_2 = 0.71 \pm 0.12 \). The input value of \( (r_V, r_2) \) for each of
Figure 5.4: $\delta m$ fits with anti-electrons (left) and with anti-muons (right) in CLEO II.V data.

Figure 5.5: $\delta m$ fits with anti-electrons (left) and with anti-muons (right) in CLEO II data.
the signal MC samples was (1.84, 0.59), (1.84, 0.71), (1.84, 0.83), (1.704, 0.71) and (1.976, 0.71). Each signal MC sample had 300,000 events.

With these signal MC samples, the $\delta m$ signal shapes for the $\delta m$ fits were obtained with the same method used in the normal analysis. The numbers of signal $D^+ \rightarrow \bar{K}^{*0}e^+\nu_e$ events for CLEO II.V data were measured by $\delta m$ fits with these $\delta m$ signal shapes. I used the same $\delta m$ background fit function with the same values of parameters because the $\delta m$ background shape parameters were determined from a background MC sample.

The efficiency corrected numbers of signal events assuming different values of $(r_V, \ r_2)$ pairs are shown in Table 5.2. The systematic error produced by the model dependence of signal MC was calculated by the quadratic sum of variation of the efficiency corrected numbers of signal events due to the different values of $(r_V, \ r_2)$ pairs and the half of the difference between the efficiency corrected numbers of signal events obtained using the ISGW2 model and those using $(r_V, \ r_2) = (1.84, 0.71)$. To obtain the efficiency corrected numbers of signal events due to the different values of $(r_V, \ r_2)$ pairs, the following equation was used:

$$
\delta N = \sqrt{\left(\frac{\partial N}{\partial r_V}\right)^2 \cdot (\delta r_V)^2 + \left(\frac{\partial N}{\partial r_2}\right)^2 \cdot (\delta r_2)^2 + 2 \cdot Cov(r_V, r_2) \cdot \left(\frac{\partial N}{\partial r_V}\right) \cdot \left(\frac{\partial N}{\partial r_2}\right) \cdot \delta r_V \cdot \delta r_2},
$$

(5.1)

where $N$ is the efficiency corrected number of signal events and $Cov(r_V, r_2)$, the correlation coefficient between $r_V$ and $r_2$, was measured as -0.123 at E791. The resulting quadratic sum, 1.0%, was used as the systematic error due to the model dependence of signal MC samples.

5.2.5 Efficiency Variation due to Fragmentation Modeling

Since the fragmentation in charm productions is not perfectly understood, there may be some level of discrepancy in fragmentation between data and MC. Even though the momentum cuts on $D$'s in the semileptonic and hadronic modes were tuned to minimize this uncertainty, it is necessary to estimate its size. This was done by estimating how much the fragmentation difference between data and MC affects the
Table 5.2: The efficiency corrected number of signal $D^+ \rightarrow \bar{K}^*0\pi^+\nu_\mu$ events for CLEO II.V data. The $\delta m$ signal shape of each measurement was obtained with signal MC with the given form factor ratios, $r_V$ and $r_2$.

<table>
<thead>
<tr>
<th>$(r_V, r_2)$</th>
<th>Eff. Corrected Number of Signal Events</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1.976, 0.71)</td>
<td>71293.2 ± 5468.8</td>
</tr>
<tr>
<td>(1.704, 0.71)</td>
<td>70773.7 ± 5367.3</td>
</tr>
<tr>
<td>(1.84, 0.83)</td>
<td>69002.3 ± 5288.9</td>
</tr>
<tr>
<td>(1.84, 0.59)</td>
<td>67811.6 ± 5192.7</td>
</tr>
<tr>
<td>(1.84, 0.71)</td>
<td>70327.3 ± 5357.9</td>
</tr>
<tr>
<td>ISGW2 Model</td>
<td>71620.7 ± 5425.5</td>
</tr>
</tbody>
</table>

efficiency measurements. The fragmentation function obtained from the hadronic mode analysis was used for this study. This study included the following steps.

1. In the $D^+ \rightarrow K^-\pi^+\pi^+$ mode, data and MC samples were divided into seven bins in $|\vec{p}_D|$ from 1.5 to 5.0 GeV.

2. Then, I figured out the scale factor which matches the fraction of MC events in each $|\vec{p}_D|$ bin to that of the data.

3. Using the scaling factors obtained in the second step, the efficiencies for both the semileptonic and the hadronic modes were recalculated. In the hadronic mode, the number of entries of each of $|\vec{p}_D|$ bin was multiplied by the scaling factor of the $|\vec{p}_D|$ bin. In the semileptonic mode, the generated $|\vec{p}_D|$ was used instead of the reconstructed $|\vec{p}_D|$ to do the same process. Based on these scaled number of entries in the hadronic and semileptonic modes, the scaled efficiencies were calculated.

4. I calculated $\left(\epsilon_{\text{org}}/\epsilon_{\text{scid}}\right)_{sl} \cdot \left(\epsilon_{\text{scid}}/\epsilon_{\text{org}}\right)_{had}$. Here, subscripts $sl$ and $had$ stand for the semileptonic and the hadronic modes, and $\epsilon_{\text{org}}$ and $\epsilon_{\text{scid}}$ are the original and scaled efficiencies. The deviation of this ratio from 1.0 was assigned to the systematic error.

Fig. 5.6 shows a comparison of the momenta of reconstructed $D$’s between data and MC. Fig. 5.7 shows the comparison of the scaled and un-scaled efficiencies. The
Figure 5.6: $D$ momentum comparison between data and MC in CLEO II.V $D^+ \rightarrow K^-\pi^+\pi^+$ (left), and CLEO II $D^+ \rightarrow K^-\pi^+\pi^+$ (right).

<table>
<thead>
<tr>
<th>mode</th>
<th>CLEO II.V</th>
<th>CLEO II</th>
</tr>
</thead>
<tbody>
<tr>
<td>semileptonic</td>
<td>0.983</td>
<td>0.975</td>
</tr>
<tr>
<td>hadronic</td>
<td>0.972</td>
<td>0.971</td>
</tr>
<tr>
<td>$(\epsilon_{\text{org}}/\epsilon_{\text{scid}})<em>{\text{sl}} \cdot (\epsilon</em>{\text{scid}}/\epsilon_{\text{org}})_{\text{had}}$</td>
<td>1.011</td>
<td>1.004</td>
</tr>
</tbody>
</table>

measured $\epsilon_{\text{org}}/\epsilon_{\text{scid}}$ is summarized in Table 5.3.

5.2.6 Feed Down

Even though there has been no branching ratio measurement for the $D^+ \rightarrow \bar{K}_2^* l^+\nu_e$ decay mode, the ISGW2 model predicts a non-zero partial width for this mode. [35] The ISGW2 model predicts that all partial widths for $D^+ \rightarrow X_{\text{ne}} e^+\nu_e$ are zero except for $K^0$, $K_{(892)}^*$ and $K_2^*$ semileptonic decays. Furthermore the predicted partial width ratio between $K_2^*$ and $K(892)^*$ semileptonic decay is 1/16. This ratio makes the branching fraction of $D^+ \rightarrow \bar{K}_2^* l^+\nu_l$ to be 0.3%, when the branching fraction of $D^+ \rightarrow \bar{K}^{*0} l^+\nu_l$ is assumed to be 4.8%. [11] If this prediction is accepted, the rate from $D^+$ to $\bar{K}^{*0} l^+\nu_l$ through $\bar{K}_2^*$ would be 0.058%
Figure 5.7: Comparison of the scaled and un-scaled efficiencies in CLEO II.V $D^+ \rightarrow K^{*0} e^+ \nu_e$ (top left), CLEO II $D^+ \rightarrow K^* e^+ \nu_e$ (top right), CLEO II.V $D^+ \rightarrow K^- \pi^+ \pi^+$ (bottom left), and CLEO II $D^+ \rightarrow K^- \pi^+ \pi^+$ (bottom right).
Since the total rate from $\bar{K}^{\ast}_{2}$ to $K^{\ast 0}X$ is 19.2%, the rate for $D^+ \rightarrow \bar{K}^{\ast 0}l^{+}\nu_l$, $\bar{K}^{\ast}_{2} \rightarrow \bar{K}^{\ast 0}X$ is about 1.2% of that of $D^+ \rightarrow \bar{K}^{\ast 0}l^{+}\nu_l$. When $D^+ \rightarrow \bar{K}^{\ast}e^{+}\nu_e$ and $\bar{K}^{\ast}_{2} \rightarrow \bar{K}^{\ast 0}X$ events were analyzed as $D^+ \rightarrow \bar{K}^{\ast}e^{+}\nu_e$ events, however, the efficiency of $D^+ \rightarrow \bar{K}^{\ast}l^{+}\nu_e$ was about 20% of that of $D^+ \rightarrow \bar{K}^{\ast 0}e^{+}\nu_e$. This efficiency reduces the above 1.2% to 0.24%, and this entire 0.24% was assigned as the systematic error due to the feed down from $D^+ \rightarrow \bar{K}^{\ast}l^{+}\nu_e$.

### 5.2.7 $D^+ \rightarrow \bar{K}^{\ast 0}\pi^{0}l^{+}\nu_l$

To estimate the contribution from the $D^+ \rightarrow \bar{K}^{\ast 0}\pi^{0}l^{+}\nu_l$ decay mode, $D^+ \rightarrow \bar{K}^{\ast 0}\pi^{0}e^{+}\nu_e$ MC events were generated using the phase space and analyzed as $D^+ \rightarrow \bar{K}^{\ast 0}e^{+}\nu_e$ signal events. The relative efficiency of $D^+ \rightarrow \bar{K}^{\ast 0}\pi^{0}e^{+}\nu_e$ MC events with respect to that of $D^+ \rightarrow \bar{K}^{\ast 0}e^{+}\nu_e$ was found to be 84%. Since the upper limit of the branching fraction of $D^+ \rightarrow \bar{K}^{\ast 0}\pi^{0}\mu^{+}\nu_\mu$ with respect to $D^+ \rightarrow K^{-}\pi^{+}\mu^{+}\nu_\mu$ is 0.042 [17] at the 90% confidence level, I assigned the product of relative efficiency and the upper limit of the branching fraction, 3.5% to the systematic error due to the contribution of $D^+ \rightarrow \bar{K}^{\ast 0}\pi^{0}l^{+}\nu_l$.

### 5.2.8 Non-resonant $D^+ \rightarrow K^{-}\pi^{+}l^{+}\nu_l$

Using the phase space, $D^+ \rightarrow K^{-}\pi^{+}e^{+}\nu_e$ MC events were generated and analyzed as $D^+ \rightarrow \bar{K}^{\ast 0}e^{+}\nu_e$ signal MC events. The efficiency of these events to be mistaken as $D^+ \rightarrow \bar{K}^{\ast 0}l^{+}\nu_l$ was zero within the statistical error of 0.5%. Therefore the systematic error due to the non-resonant $D^+ \rightarrow K^{-}\pi^{+}l^{+}\nu_l$ decay was neglected.

### 5.2.9 Uncertainties in Normalization Mode Analysis

The uncertainties in the signal and background shapes in $\delta m$ fits in the normalization mode analyses were estimated separately. To estimate the uncertainty in the signal shape, the width of $M_{D^+}$ was floated in $M_{D^+}$ fits. The quadratic difference between the errors of the efficiency-corrected yields when the $M_{D^+}$ width was floated and fixed was used as the systematic error in the signal shape. The estimated values were 1.8%
and 2.8% for CLEO II.V and CLEO II, respectively.

The appropriateness of using the $\delta m$ background function of equation 4.1 was studied as the source of systematics related to the $\delta m$ background shape. To estimate this systematic error, the $\delta m$ histogram in Fig. 4.3 obtained using CLEO II.V $\pi^0$ mass sideband data was used as a $\delta m$ background fit function instead of the function in equation 4.1. The half of the difference between the efficiency-corrected yield in this method and that in the normal analysis, 1.1%, was used as the systematic error for both CLEO II.V and CLEO II.

5.2.10 Track Finding Efficiency

The systematic error related to track finding efficiency was ignored. The number of final charged tracks are the same in both of $D^+ \rightarrow \bar{K}^{*0} l^+ \nu_l$ and $D^+ \rightarrow K^- \pi^+ l^+ \nu_l$ mode. When leptons were identified, the same track quality cuts used for other charged tracks were applied. As a result, the systematics in track finding efficiency cancels out between the semi-leptonic and the hadronic modes assuming that the systematics in lepton track finding efficiency are comparable with those of charged $\pi$ or $K$ tracks.

5.2.11 Summary of Systematic Error Estimation

The contribution from each systematic error source is given in Table 5.4. The systematic errors for each source were estimated separately for the electron and muon modes in both CLEO II.V and CLEO II data.

To figure out the systematic errors for the electron (muon) mode, CLEO II.V and CLEO II electron (muon) mode systematic errors were combined. Then, the combined electron and muon mode systematic errors were combined again to obtain the lepton mode systematic errors. The combined results are given in Table 5.5. The details of this combining method are described in APPENDIX A.
Table 5.4: Systematic Error Estimation. The unit of the values is $10^{-2}$. The percentages of the values are expressed in parentheses. For the estimation of values, refer Table 5.6.

<table>
<thead>
<tr>
<th>source</th>
<th>CLEO II.V</th>
<th>CLEO II</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\epsilon$ mode</td>
<td>$\mu$ mode</td>
</tr>
<tr>
<td>fits</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta m$ sig</td>
<td>2.7 (3.9%)</td>
<td>4.2 (6.3%)</td>
</tr>
<tr>
<td>$\delta m$ bgrnd</td>
<td>1.3 (1.9%)</td>
<td>1.9 (2.8%)</td>
</tr>
<tr>
<td>data MC diff</td>
<td>3.1 (4.5%)</td>
<td>3.0 (4.5%)</td>
</tr>
<tr>
<td>lepton id</td>
<td>1.4 (2.0%)</td>
<td>0.34 (0.3%)</td>
</tr>
<tr>
<td>fake leptons</td>
<td>0.082 (0.12%)</td>
<td>0.046 (0.11%)</td>
</tr>
<tr>
<td>sig MC model dep</td>
<td>0.68 (1.0%)</td>
<td>0.67 (1.0%)</td>
</tr>
<tr>
<td>eff. by fragmnt.</td>
<td>0.75 (1.1%)</td>
<td>0.74 (1.1%)</td>
</tr>
<tr>
<td>feed down</td>
<td>0.16 (0.24%)</td>
<td>0.16 (0.24%)</td>
</tr>
<tr>
<td>$D^+ \rightarrow K^{*0}\pi^0\ell^+\nu_\ell$</td>
<td>2.4 (3.5%)</td>
<td>2.3 (3.5%)</td>
</tr>
<tr>
<td>normal. mode</td>
<td></td>
<td></td>
</tr>
<tr>
<td>sig. shape</td>
<td>1.2 (1.8%)</td>
<td>1.2 (1.8%)</td>
</tr>
<tr>
<td>bgr. shape</td>
<td>0.75 (1.1%)</td>
<td>0.74 (1.1%)</td>
</tr>
<tr>
<td>TOTAL</td>
<td>6.1</td>
<td>6.8</td>
</tr>
</tbody>
</table>

Table 5.5: Combined systematic errors. The $\epsilon$ and $\mu$ mode columns contain combined CLEO II.V and CLEO II systematic errors. Combined electron and muon mode systematic errors are given in the $l$ column. The unit of the values is $10^{-2}$. For details on the combining method, refer to APPENDIX A.

<table>
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<tr>
<th>source</th>
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<th>$l$ mode</th>
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<tr>
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<tr>
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</tr>
<tr>
<td>$\delta m$ bgrnd</td>
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</tr>
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<td>2.6</td>
</tr>
<tr>
<td>lepton id</td>
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<td>0.89</td>
</tr>
<tr>
<td>fake leptons</td>
<td>0.064</td>
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<td>0.17</td>
</tr>
<tr>
<td>sig MC model dep</td>
<td>0.74</td>
<td>0.72</td>
<td>0.74</td>
</tr>
<tr>
<td>efficiency by fragmnt.</td>
<td>0.53</td>
<td>0.54</td>
<td>0.53</td>
</tr>
<tr>
<td>feed down</td>
<td>0.18</td>
<td>0.17</td>
<td>0.18</td>
</tr>
<tr>
<td>$D^+ \rightarrow K^{*0}\pi^0\ell^+\nu_\ell$</td>
<td>2.6</td>
<td>2.5</td>
<td>2.6</td>
</tr>
<tr>
<td>normalization mode</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>sig. shape</td>
<td>1.1</td>
<td>1.1</td>
<td>1.1</td>
</tr>
<tr>
<td>bgr. shape</td>
<td>0.81</td>
<td>0.79</td>
<td>0.81</td>
</tr>
<tr>
<td>TOTAL</td>
<td>4.8</td>
<td>5.4</td>
<td>4.6</td>
</tr>
</tbody>
</table>
Table 5.6: Results of $R_l^+$ measurements with statistical errors.

<table>
<thead>
<tr>
<th></th>
<th>CLEO II.V</th>
<th>CLEO II</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_e^+$</td>
<td>0.68 ± 0.05</td>
<td>0.87 ± 0.07</td>
</tr>
<tr>
<td>$R_\mu^+$</td>
<td>0.67 ± 0.12</td>
<td>0.84 ± 0.20</td>
</tr>
</tbody>
</table>

Table 5.7: Results of $R_e^+$, $R_\mu^+$ and $R_l^+$ measurements with the CLEO II.V and CLEO II data.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_e^+$</td>
<td>0.74 ± 0.04(stat.) ± 0.05(sys.)</td>
</tr>
<tr>
<td>$R_\mu^+$</td>
<td>0.72 ± 0.10(stat.) ± 0.05(sys.)</td>
</tr>
<tr>
<td>$R_l^+$</td>
<td>0.74 ± 0.04(stat.) ± 0.05(sys.)</td>
</tr>
</tbody>
</table>

5.3 Calculation of $R_l$

From the results of the fits, the branching fraction ratios of $D^+ \rightarrow \bar{K}^*0^+ \nu_l$ with respect to $D^+ \rightarrow K^-\pi^+\pi^+$ were calculated using the equation below:

$$R_l^+ = \frac{N_{sl}}{\epsilon_{sl} \cdot B(\bar{K}^*0^+ \rightarrow K^-\pi^+)} \cdot \frac{\epsilon_{had}}{N_{had}}, \tag{5.2}$$

They are presented in Table 5.6. In equation 5.2, $N_{sl}$ and $\epsilon_{sl}$ are the observed number of events and the efficiency for $D^+ \rightarrow \bar{K}^*0^+ \nu_l$, respectively. $N_{had}$ and $\epsilon_{had}$ are the same quantities for $D^+ \rightarrow K^-\pi^+\pi^+$.

The measured branching fraction ratios including their errors are summarized in Table 5.7. The combined $R_l^+$ is also given in Table 5.7. My method for combining $R_l^+$’s is described in APPENDIX A.

Table 5.7 shows an excellent agreement between $R_e^+$ and $R_\mu^+$. However, $R_e^+$’s of CLEO II.V and CLEO II are about 1.5$\sigma$ apart from each other. APPENDIX E describes how I attempted to show that this difference is merely statistical fluctuation, and not a systematic problem.
CHAPTER 6
ANALYSIS METHOD CHECKS

To check my overall analysis method and results, nine tests were done:

1. Use only $\tilde{p}_{\nu 1}$ instead of using the best of three solutions,
2. Use pseudo $\delta m$ instead of $\delta m$,
3. Analyze a continuum MC sample with the normal analysis method,
4. Analyze with the $\delta m$ signal shape obtained with QQ tagged signal MC events,
5. Analyze data with a $\pi^0$ mass sideband subtraction,
6. Change the order of the fits: execute the $\delta m$ fits first and do the $K\pi$ mass fit afterward,
7. Use a $M_{K\pi}$ cut of $0.87 < M_{K\pi} < 0.92$ GeV instead of the $K\pi$ mass fit,
8. Use the fixed parameters of the $K\pi$ mass background fit function extracted from generic continuum MC background events, and
9. Analyze data in the $D^+ \rightarrow K^-\pi^+\pi^+$ mode with the full reconstruction and the “$\nu$ reconstruction” method. One of the $\pi^+$’s is reconstructed with the “$\nu$ reconstruction” method. Then, compare the efficiency-corrected yields with those of full reconstruction method.

6.1 Analysis Using $\tilde{p}_{\nu 1}$

Since the error obtained using $\tilde{p}_{\nu 1}$ (see Fig. 3.1) is comparable to the error of the normal analysis (see Table B.1 in APPENDIX B), it would give a good cross check whether these two methods give consistent results. In this analysis, everything was the same as the normal analysis except:
Figure 6.1: $\delta m$ fits for the electron mode in CLEO II.V (left) and CLEO II data (right). The $\delta m$ histograms were obtained using $\vec{p}_{e1}$ only.

- The $\delta m$ calculated from $\vec{p}_{e1}$ was always used.
- The background $\delta m$ histograms from generic continuum MC were used as the background function in the $\delta m$ fits. To obtain these background $\delta m$ histograms all events containing signal decay mode were discarded. Then a $M_{K\pi}$ cut, $0.87 < M_{K\pi} < 0.92$ GeV, was applied.

$R_e$’s and $R_{\mu}$’s calculated for the CLEO II.V and CLEO II data with this method are $0.63 \pm 0.05$ and $0.63 \pm 0.13$, respectively for the electron and muon mode for the CLEO II.V data and $0.89 \pm 0.07$ and $0.74 \pm 0.19$, respectively for those of the CLEO II data. Fig. 6.1 shows the $\delta m$ fit results for the electron mode in the CLEO II.V and CLEO II data. The $\delta m$ fit results for the muon mode in the CLEO II.V and CLEO II data are shown in Fig. 6.2.

## 6.2 Analysis Using Pseudo $\delta m$

The previous CLEO measurement for the branching fraction of $D^+ \rightarrow \bar{K}^*0 l^+ \nu_l$ used pseudo $\delta m$ rather than $\delta m$. To check the consistency of the analysis results as well
Figure 6.2: $\delta m$ fits for the muon mode in CLEO II.V (left) and CLEO II data (right). The $\delta m$ histograms were obtained using $\bar{p}_{\nu1}$ only.

as to quantify how much gain $\delta m$ gives, the analysis was repeated with pseudo $\delta m$. The same analysis cuts and the same analysis method were used except that the data was divided into 50 pseudo $\delta m$ bins from 0.135 GeV to 0.235 GeV. For each of pseudo $\delta m$ bins, the number of $K^*$'s was extracted with the $K\pi$ mass fit with the same signal and background fit functions as in the normal analysis. After plotting these numbers of $K^*$'s as a function of pseudo $\delta m$, the number of signal events was obtained with a pseudo $\delta m$ fit. In this pseudo $\delta m$ fit, the signal shape was extracted from the signal MC like in the normal analysis. For the background fit function, the following continuous function was used.

\[
\begin{align*}
A_{bkg} \times (\delta m_{ps} - 0.135) Q_{pdm} & \quad ; \quad 0.135 \leq \delta m_{ps} < 0.1412\text{GeV}, \\
A_{bkg} \times (\delta m_{ps} - 0.135) Q_{pdm} e^{-B_{pdm}(\delta m_{ps} - 0.1412)} & \quad ; \quad 0.1412 \leq \delta m_{ps} < 0.235\text{GeV},
\end{align*}
\]

(6.1)

where $\delta m_{ps}$ in the equation 6.1 means pseudo $\delta m$.

Like in the normal analysis, the values of $Q_{pdm}$ and $B_{pdm}$ in the above background fit function were determined with a continuum MC sample. Once these parameters were obtained, the values were fixed in the pseudo $\delta m$ fit to data.

Only $R_{\bar{c}+}$ were measured with the CLEO II.V and CLEO II data. The measured
$R_c^+$'s are $0.67 \pm 0.06$ and $0.82 \pm 0.08$ for CLEO II.V and CLEO II, respectively (refer to table 6.1.). These $R_c^+$'s are consistent with those with the $\delta m$. However, their statistical errors are about 15% and 21% larger than those obtained with the $\delta m$ analysis for CLEO II.V and CLEO II, as described in section 3.3. The pseudo $\delta m$ fits for CLEO II.V and CLEO II are shown in Fig. 6.3.

6.3 Analyzing Continuum MC Events

Analyzing a continuum MC sample is a straightforward way to check the analysis method and results since I know that the input value of $R_c^+$ is 0.56 [36]. Fig. 6.4 shows the $\delta m$ fit results for $D^+ \rightarrow \bar{K}^*0 e^+ \nu_e$ with 17 million CLEO II.V continuum MC events.

Fig. 6.5 shows the $\delta m$ fit results for the same continuum MC events in $D^+ \rightarrow K^- \pi^+ \pi^+$ mode. As in the data analysis, the $\delta m$ plot was made with $M_{D^+}$ fits in the $\delta m$ bins.

The measured $R_c^+$ was $0.53 \pm 0.07$. This result is consistent with the input value of $R_c^+$. 

Figure 6.4: $\delta m$ fit results with 17 million CLEO II.V continuum MC events in $D^+ \rightarrow \bar{K}^*\pi^+\pi^+\nu_e$ mode.

Figure 6.5: $\delta m$ fit results with 17 million CLEO II.V continuum MC events in $D^+ \rightarrow K^-\pi^+\pi^+$ mode.
6.4 Analysis Using QQ Tagged $\delta m$ Signal Shape

In the normal analysis, all combinations from signal MC events which passed analysis cuts were regarded as signal. As a result, the signal shape on a $\delta m$ histogram has not only peak but also a long tail. Instead of this $\delta m$ signal shape, the $\delta m$ distribution of only right combinations tagged by QQ information was used as the signal shape. Since only QQ tagged signal events are considered as signal, a $\delta m$ histogram made with a signal MC sample contains background. To describe the background from signal events, an additional $\delta m$ background function

\[
\begin{align*}
A_{bkg} \times (\delta m - 0.135)^{Q_{sl}} & \quad ; \quad 0.135 \leq \delta m < 0.1412 \quad \text{GeV}, \\
A_{bkg} \times (\delta m - 0.135)_{Q_{sl}} e^{-B_{sl}(\delta m - 0.1412)} & \quad ; \quad 0.1412 \leq \delta m < 0.235 \quad \text{GeV}.
\end{align*}
\]

(6.2)

was used. Efficiency was calculated using a $\delta m$ fit of “signal” and “background” above to signal MC distribution. All parameters in the $\delta m$ fit were floated.

The $\delta m$ fit to data was executed with three fit functions. The signal, background and background in the signal events. These two backgrounds were described with the function in equation 6.2 with different values of parameters. In the data $\delta m$ fit, the relative area of the signal with respect to that of the background of signal events was constrained to the ratio obtained with a signal MC sample which was statistically independent from the signal MC sample used for the efficiency measurement. Furthermore, the values of the shape parameters of the background of signal events and the background of background events, $Q_{sl}$’s and $B_{sl}$’s, were fixed to the values measured with the above signal MC sample and a generic continuum MC events excluding signal decay mode, respectively.

Only $R_e$’s were measured for CLEO II and CLEO II.V data. The efficiencies were 2.68% and 2.66% for CLEO II.V and CLEO II, respectively. The CLEO II.V and CLEO II $R_e$’s were $0.69 \pm 0.05$ and $0.82 \pm 0.08$, respectively (refer to table 6.1.), and these values are in good agreement with $R_e$’s in Table 5.6. Fig. 6.6 shows the $\delta m$ fits for the electron mode of CLEO II.V and CLEO II data, respectively.
Figure 6.6: $\delta m$ fits for the electron mode in the CLEO II.V (left) and CLEO II data (right) with the QQ tagged $\delta m$ signal shapes.

### 6.5 Analysis Using $\pi^0$ Mass Sideband Subtraction

Since a $\pi^0$ mass sideband subtraction alters the background shape in the $M_{K\pi}$ and the $\delta m$ histograms, analyzing data with a $\pi^0$ mass sideband subtraction will be a good test of this analysis. In the $\pi^0$ mass sideband subtraction, the signal band was chosen to be $|M_{\gamma\gamma} - M_{\pi^0}| < 2.5\sigma$ and the region satisfying $3.0\sigma < |M_{\gamma\gamma} - M_{\pi^0}| < 5.5\sigma$ was considered as the sideband. For each $\delta m$ bin, this $\pi^0$ mass sideband subtraction was executed and a $K\pi$ mass histogram was made. Then, the $K\pi$ mass fit was done.

The signal fit function in equation 3.1 was used for the signal part of the $K\pi$ mass fit. The values of parameters of this signal function were kept the same as those in the normal analysis. As in the normal analysis, the background function of equation 3.3 with floating parameters was used for describing the background in the fit. Then, a histogram of the number of $K^*$'s versus $\delta m$ was made. In the $\delta m$ fit, the signal and the background shape were determined with a signal and a generic continuum MC sample as in the normal analysis. The measured $R_\pi^+$'s for CLEO II.V and CLEO II data were $0.58 \pm 0.09$ and $0.86 \pm 0.11$, respectively (refer to table 6.1.). These $R_\pi^+$'s agree well with those in Table 5.6. The results of the $\delta m$ fits for $D^+ \rightarrow \bar{K}^{\ast 0} e^+ \nu_e$ and
Figure 6.7: The $\delta m$ fit in CLEO II.V $D^+ \rightarrow \bar{K}^*0 e^+ \nu_e$ analysis with $\pi^0$ mass sideband subtraction.

for $D^+ \rightarrow K^- \pi^+ \pi^+$ for CLEO II.V data are given in Fig. 6.7 and Fig. 6.8, respectively.

### 6.6 Changing Fit Order

In the normal analysis, the $K\pi$ mass fits were executed first, and the $\delta m$ fit was second. Changing the order of these fits, however, should yield the same results if the signal and background shapes are adequate.

The $M_{K\pi}$ window from 0.64 to 1.24 GeV was divided into 20 equally spaced bins. For each of these $M_{K\pi}$ bins, a $\delta m$ fit was executed. The signal shapes of these $\delta m$ fits were obtained with signal MC events and $\delta m$ histograms made from continuum MC background events were used as the $\delta m$ background fit functions. There were 25 bins in the $\delta m$ histograms in the $\delta m$ window from 0.135 to 0.235 GeV. The numbers of signal combinations extracted from these $\delta m$ fits were plotted as a function of $M_{K\pi}$. A $K\pi$ mass fit was executed with this $M_{K\pi}$ plot. The p-wave $K^*$ Breit-Wigner distribution in equation 3.1 was used as the signal fit function. The same threshold function in equation 3.3 with floating parameters was used to describe the background
Figure 6.8: The $\delta m$ fit in CLEO II.V $D^+ \rightarrow K^- \pi^+ \pi^+$ analysis with $\pi^0$ mass sideband subtraction.

in the $K\pi$ mass fit. The resulting $R_e$'s for CLEO II.V and CLEO II were $0.58 \pm 0.05$ and $0.80 \pm 0.12$, respectively. These $R_e$ values agree with those in the normal analysis method. The $M_{K\pi}$ resonance fits for CLEO II.V and CLEO II are shown in Fig. 6.9.

### 6.7 Analysis Using $M_{K\pi}$ Cut

In this method, a $0.87 < M_{K\pi} < 0.92$ GeV cut was used instead of the $K\pi$ mass fit. The numbers of entries passing the $M_{K\pi}$ cut on $M_{K\pi}$ histograms were plotted as a function of $\delta m$ and a $\delta m$ fit was used to measure the number of reconstructed events. In the $\delta m$ fit, the signal shape was extracted with signal MC events like in the normal analysis. The $\delta m$ histogram made with background events in a continuum MC sample was used as the $\delta m$ background fit function.

One of the weakness of this method is that it depends heavily on MC. The large backgrounds in the $\delta m$ plots are estimated using continuum MC. All backgrounds passing the $M_{K\pi}$ cut were included. In the normal analysis, on the other hand, the small backgrounds in the $\delta m$ histogram were mostly coming from the combinations of
two random photons. Other backgrounds were efficiently subtracted by the $K\pi$ mass fits. As a result, the effect of the dependence on MC to describe the background shape in the $\delta m$ histogram is small in the normal analysis (see Fig. 3.11 and Fig. 3.12).

The $R_e$ and $R_\mu$ in this analysis are given by $0.60 \pm 0.04$ and $0.59 \pm 0.08$ for CLEO II.V data and $0.74 \pm 0.05$ and $0.82 \pm 0.14$ for CLEO II data, respectively (refer to table 6.1.). These $R_e$’s are about one $\sigma$ smaller than those in the normal analysis. However, the $R_\mu$’s show good agreement with the normal analysis $R_\mu$’s.

Fig. 6.10 shows the $\delta m$ fit results with this method for CLEO II.V and CLEO II data.

### 6.8 Fixing $M_{K\pi}$ Background Function Parameters

In this method, the parameters in the background function of the $K\pi$ mass fits were fixed to the values obtained with continuum MC background events. The $\delta m$ window from 0.135 to 0.235 GeV was divided into 25 equally sized bins. For each of the $\delta m$ bins, a $K\pi$ mass fit was executed with the background function from the $M_{K\pi}$ fit in equation 3.3. After obtaining the values of these parameters, they were fixed
Figure 6.10: $\delta m$ fits with the $M_{K\pi}$ cut method in $D^+ \rightarrow K^{*0} e^+ \nu_e$ for CLEO II.V data (upper left), in $D^+ \rightarrow K^{*0} \mu^+ \nu_\mu$ for CLEO II.V data (upper right), in $D^+ \rightarrow K^{*0} e^+ \nu_e$ for CLEO II data (lower left), and in $D^+ \rightarrow K^{*0} \mu^+ \nu_\mu$ for CLEO II.V data (lower right).
Figure 6.11: \( \delta m \) fits with fixed parameters in the \( M_{K\pi} \) background fit function in \( D^+ \to K^{*0}e^+\nu_e \) in the CLEO II.V (left) and CLEO II data (right).

in the \( M_{K\pi} \) fits for measuring \( R_e \)'s or \( R_\mu \)'s with data. Like the \( M_{K\pi} \) cut method in section 6.7, this method yields results strongly dependent on MC since all background shapes in this method were extracted from continuum MC events.

With this method, only \( R_e \)'s were measured for CLEO II.V and CLEO II data. They are \( 0.60 \pm 0.04 \) and \( 0.77 \pm 0.06 \), respectively. The measured \( R_e \)'s were about one \( \sigma \) smaller than those in the normal analysis. Fig. 6.11 shows the \( \delta m \) fit results from this method for CLEO II.V and CLEO II data.

6.9 Analysis of \( D^+ \to K^-\pi^+\pi^+ \) Mode Using \( \nu \) Reconstruction

Using \( D^+ \to K^-\pi^+\pi^+ \) mode, the “\( \nu \) reconstruction” method which this analysis used was tested focusing on two issues: (i) whether this \( \nu \) reconstruction method gives the right results in data, and (ii) how the \( \nu \) reconstruction method alters the signal and the background shape in the \( \delta m \) histogram. In this analysis, one of the \( \pi^+ \)'s was assumed to be undetected and its momentum vector was reconstructed using the “\( \nu \) reconstruction” method. To check whether the “\( \nu \) reconstruction” method gives the right results, the \( N/\epsilon \) obtained with this method was compared with that of the full
Figure 6.12: $\delta m$ fits in the CLEO II.V $D^+ \rightarrow K^-\pi^+\pi^+$ analysis with the full reconstruction (left) and the “$\nu$ reconstruction” method (right).

$D^+ \rightarrow K^-\pi^+\pi^+$ reconstruction. Here $N$ is the detected number of events and $\epsilon$ is the efficiency. All CLEO II.V data were used for this study.

To obtain the raw yields and efficiencies, the method described in section 4.2 in chapter 4 was used. In this comparison, however, a $0.135 \leq \delta m \leq 0.235$ $\delta m$ window with 2.5 MeV bin width was used since the $\delta m$ peak in the “$\nu$ reconstruction” method is not as narrow as that in the full reconstruction. It may sound strange to use $M_{D^+}$ fits in this method since the “$\nu$ reconstruction” constrains $M_{K\pi\pi}$ to $M_{D^+}$, 1.869 GeV. The distributions of $M_{D^+}$ used in the $M_{D^+}$ fits were distributions of $K^-\pi^+\pi^+$ mass even though the $\delta m$ was obtained with the “$\nu$ reconstruction” method. The numbers of detected events were $N_{full} = 25246.0 \pm 406.3$ with the full reconstruction and $N_\nu = 24726.0 \pm 905.8$ with the “$\nu$ reconstruction” method. The efficiencies were 15.6% and 14.9% for the full reconstruction and the “$\nu$ reconstruction” methods, respectively. $(N/\epsilon)_{full}/(N/\epsilon_\nu)$ was, then, calculated to be $0.97 \pm 0.05$. Fig. 6.12 shows the $\delta m$ fit results of the full reconstruction and the “$\nu$ reconstruction” method.

The normalization mode analysis using the “$\nu$ reconstruction” method has one advantage compared to the semileptonic mode analysis. In the normalization mode, $M_{D^+}$ fits were used to make a $\delta m$ histogram. As a result, the background in a nor-
Figure 6.13: $\delta m$ fit in the CLEO II.V $D^+ \rightarrow K^-\pi^+\pi^+$ analysis with the “$\nu$ reconstruction” method with $M_{D^+}$ cut.

The background shape in Fig. 6.13 was described by the $\delta m$ background function for $D^+ \rightarrow K^-\pi^+\pi^+$ in equation 4.1. The values of the background shape parameters were floated in the fits.

Fig. 6.14 shows how much the “$\nu$ reconstruction” alters the overall (left) and the background (right) $\delta m$ shapes. The $\delta m$ plots in Fig. 6.14 were obtained with a $M_{D^+}$ cut, $1.84 \leq M_{D^+} \leq 1.90$ GeV instead of $M_{D^+}$ fits. Even though one would expect
Figure 6.14: Comparison of over all $\delta m$ (left) and background $\delta m$ distributions (right) made with full reconstruction and with the “$\nu$ reconstruction” method in $D^+ \rightarrow K^-\pi^+\pi^+$. These plots were made with a CLEO II.V continuum MC sample.

that “$\nu$ reconstruction” would bias the $\delta m$ background shape significantly since the $\nu$ momentum giving the $\delta m$ value closest to the nominal $D^*-D$ mass difference was chosen among the three $\nu$ momentum estimates, the $\delta m$ background shape was not changed much.

Fig. 6.15 shows the distribution of $|\vec{p}_D|$ types from the “$\nu$ reconstruction” method in $D^+ \rightarrow K^-\pi^+\pi^+$. Since there are no undetectable final particles in $D^+ \rightarrow K^-\pi^+\pi^+$, the missing momentum of an event was not considered for the $\nu$ momentum estimation.

6.10 Summary of Analysis Check

Analyzing a continuum MC sample with the normal analysis method gave an excellent agreement with the input value of $R_0$.

The $D^+ \rightarrow K^-\pi^+\pi^+$ mode was analyzed with a “$\nu$ reconstruction” method in which the momentum of the one of the $\pi^+$’s was reconstructed. The efficiency-corrected number of detected events with this method was very consistent with that
Figure 6.15: Distribution of $|\bar{p}_D|$ types in the “$\nu$ reconstruction” of $D^+ \rightarrow K^-\pi^+\pi^+$.

with the full reconstruction.

$R_e$ and $R_\mu$ were measured with various methods and the results are shown in Table 6.1. All of these method gave $R_e^+$’s and $R_\mu^+$’s that are consistent within errors.
Table 6.1: Results of $R_i^+$ measurements with various analysis methods. Only statistical errors are presented.

<table>
<thead>
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<th>method</th>
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<th></th>
<th>CLEO II</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R_e^+$</td>
<td>$R_\mu^+$</td>
<td>$R_e^+$</td>
<td>$R_\mu^+$</td>
</tr>
<tr>
<td>$\delta m$ with most favored $\bar{\nu}$</td>
<td>0.63 ± 0.05</td>
<td>0.63 ± 0.13</td>
<td>0.89 ± 0.07</td>
<td>0.74 ± 0.19</td>
</tr>
<tr>
<td>pseudo $\delta m$</td>
<td>0.67 ± 0.06</td>
<td></td>
<td>0.82 ± 0.08</td>
<td></td>
</tr>
<tr>
<td>QQ tagged $\delta m$ signal shape</td>
<td>0.69 ± 0.05</td>
<td></td>
<td>0.82 ± 0.11</td>
<td></td>
</tr>
<tr>
<td>$M_{\pi 0}$ sideband subt.</td>
<td>0.58 ± 0.09</td>
<td></td>
<td>0.86 ± 0.11</td>
<td></td>
</tr>
<tr>
<td>change order of fits</td>
<td>0.58 ± 0.05</td>
<td></td>
<td>0.80 ± 0.12</td>
<td></td>
</tr>
<tr>
<td>$M_{K\pi}$ cut</td>
<td>0.60 ± 0.04</td>
<td>0.59 ± 0.08</td>
<td>0.74 ± 0.05</td>
<td>0.82 ± 0.14</td>
</tr>
<tr>
<td>fixed bgr. par. in $M_{K\pi}$ fit</td>
<td>0.60 ± 0.04</td>
<td>0.59 ± 0.08</td>
<td>0.77 ± 0.06</td>
<td></td>
</tr>
<tr>
<td>Normal Analysis</td>
<td>0.68 ± 0.05</td>
<td>0.67 ± 0.12</td>
<td>0.87 ± 0.07</td>
<td>0.84 ± 0.20</td>
</tr>
</tbody>
</table>
CHAPTER 7
CONCLUSIONS

As a conclusion of this work, I will present three things:

1. a summary of the branching ratios $R_c^+$, $R_\mu^+$ and $R_l^+$ and the branching fractions $\mathcal{B}(D^+ \to \bar{K}^{*0}e^+\nu_e)$, $\mathcal{B}(D^+ \to \bar{K}^{*0}\mu^+\nu_\mu)$, and $\mathcal{B}(D^+ \to \bar{K}^{*0}\ell^+\nu_\ell)$,

2. a discussion of the impact of this work to the values of the form factors $A_1$, $A_2$ and $V$, and

3. a guideline for quark models.

7.1 Summary of Measurements

The branching ratios $R_c^+$, $R_\mu^+$ and $R_l^+$ are measured to be

$$R_c^+ = 0.74 \pm 0.04(stat.) \pm 0.05(sys.),$$

$$R_\mu^+ = 0.72 \pm 0.10(stat.) \pm 0.05(sys.)$$

and

$$R_l^+ = 0.74 \pm 0.04(stat.) \pm 0.05(sys.).$$

My values of $R_c^+$ and $R_\mu^+$ are consistent each other. The previous CLEO measurement of $R_l^+$ was $0.67 \pm 0.09 \pm 0.07$. [13] The average particle data group values are $0.54 \pm 0.05$ and $0.53 \pm 0.06$ for $R_c^+$ and $R_\mu^+$, respectively.

Fig. 7.1 shows the comparison of the results of this work, the previous measurements and the PDG averages. In this comparison, a few things should be emphasized. In $R_c^+$ measurements, the E691 measurement and $R_c^+$ of this work are the two most significant measurements. However, they differ by approximately $3\sigma$’s. To
investigate whether this difference was produced by the signal MC model difference, $D^+ \to \bar{K}^{*0}e^+\nu_e$ signal MC events were generated using the WSG model [38] used by E691. A generated-level $q^2$ (the invariant mass squared of the virtual $W$ boson) distribution and the lepton momentum spectrums in the lab frame and in the $D$ rest frame were compared with those of the ISGW2 model. All of these comparisons shown in Fig. 7.2 and in Fig. 7.3 gave consistent results. As a result, the difference in the signal MC models does not explain the difference in $R^+_e$ values. Since it is very difficult to reproduce an experimental environment similar to that of E691, further effort to understand the difference in $R^+_e$ values of this work and E691 was not made.

To provide a meaningful constraint for determining $|V_{ub}|$ (a constraint containing a comparable uncertainty to that of the current PDG value, 30\%, [11]) this discrepancy must be resolved. For this, a more precise measurement of $R^+_e$ (or $B(D^+ \to \bar{K}^{*0}l^+\nu_l)$) using larger data samples (i.e., data samples of fixed target experiments such as FOCUS or present $B$ factories) is needed. In the future, if the proposed CLEO-c experiment is approved, the statistical error in $B(D^+ \to \bar{K}^{*0}l^+\nu_l)$ could be reduced to less than 1\%. If this is accomplished, it would be possible for $B(D^+ \to \bar{K}^{*0}e^+\nu_e)$ measurements to provide a useful information to determine $|V_{ub}|$.

Using the PDG value of $B(D^+ \to K^-\pi^+\pi^+) = 9.0 \pm 0.6\%$, [11] my measurements imply that

$$B(D^+ \to \bar{K}^{*0}e^+\nu_e) = (6.7 \pm 0.4(stat.) \pm 0.5(sys.) \pm 0.4)\%,$$

$$B(D^+ \to \bar{K}^{*0}\mu^+\nu_\mu) = (6.5 \pm 0.9(stat.) \pm 0.5(sys.) \pm 0.4)\%$$

and

$$B(D^+ \to \bar{K}^{*0}l^+\nu_l) = (6.7 \pm 0.4(stat.) \pm 0.5(sys.) \pm 0.4)\%$$

where the third error is due to the error in $B(D^+ \to K^-\pi^+\pi^+)$.

Fig. 7.4 shows the comparison of the decay rate for $D^+ \to \bar{K}^{*0}e^+\nu_e$ measured in this work with those of theoretically predicted.
Figure 7.1: Schematic diagram of the measured values of $R^+_e$ and $R^+_\mu$. The error bars denote the combined values of the statistical and systematic errors.
Figure 7.2: Comparison of $q^2$ (the invariant mass of the virtual W boson) distributions of $D^+ \rightarrow \bar{K}^{*0}e^+\nu_e$ signal MC samples generated with the WSB model and the ISGW2 model.
7.2 Impact on Form Factors

The most precise measurement of the ratios among the form factors $A_1$, $A_2$ and $V$ for $D^+ \rightarrow \bar{K}^*0e^+\nu_e$ decays so far was obtained by E791. Their values at $q^2 = 0$ and $q^2_{\text{max}}$ are shown in Table 7.1. E791 used the PDG value of $B(D^+ \rightarrow \bar{K}^*0e^+\nu_e) = (4.8 \pm 0.05)%$ [11] to obtain these form-factor values at $q^2 = 0$. Then, they used the single-pole dominance assumption to extrapolate the values up to $q^2_{\text{max}}$. Since the value of $B(D^+ \rightarrow \bar{K}^*0e^+\nu_e)$ measured by this work is substantially larger than the PDG value (about 27%, or 2.8σ larger), the values of the form factors will be also significantly altered if my result is applied. The values of $A_1$, $A_2$ and $V$ obtained using my results are shown along with other theoretical predictions in Table 7.2.

If I want to understand the decay dynamics in $B$ meson semileptonic decays using knowledge of $D^+ \rightarrow \bar{K}^*0e^+\nu_e$, I need the values of the form factors themselves. In this case, without reducing the error in $R^+$, the effort to reduce the errors in the form factor ratios would be meaningless.
Figure 7.4: Comparison of the decay rate for $D^+ \rightarrow K^{*0}e^+\nu_e$ measured in this work with those of theoretically predicted by APE [39], Wuppertal [40], UKQCD [41], ELC [42] and the ISGW2 model [35]. This plot also shows the PDG value of the $D^+ \rightarrow K^{*0}e^+\nu_e$ decay rate. All values are represented in units of $10^{10}|V_{cs}|^2$ sec$^{-1}$.

Table 7.1: Measurements of $A_1, A_2$ and $V$ of $D^+ \rightarrow K^{*0}e^+\nu_e$ at E791.

<table>
<thead>
<tr>
<th>$q^2$</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$V$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q^2 = 0$</td>
<td>0.58 ± 0.03</td>
<td>0.41 ± 0.06</td>
<td>1.06 ± 0.09</td>
</tr>
<tr>
<td>$q^2 = q^2_{max}$</td>
<td>0.68 ± 0.04</td>
<td>0.48 ± 0.08</td>
<td>1.35 ± 0.12</td>
</tr>
</tbody>
</table>
Table 7.2: Theoretically predicted values of $A_1$, $A_2$ and $V$ and the modified values from the E791 measurements using the results of this work instead of the PDG value of $\mathcal{B}(D^+ \rightarrow \bar{K}^*0 e^+ \nu_e)$.

<table>
<thead>
<tr>
<th>Group</th>
<th>$A_1(0)$</th>
<th>$A_2(0)$</th>
<th>$V(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modified E791</td>
<td>0.69 ± 0.03</td>
<td>0.50 ± 0.07</td>
<td>1.24 ± 0.10</td>
</tr>
<tr>
<td>APE [39]</td>
<td>0.67 ± 0.11</td>
<td>0.49 ± 0.34</td>
<td>1.08 ± 0.22</td>
</tr>
<tr>
<td>Wuppertaal [40]</td>
<td>0.61±0.11</td>
<td>0.83±0.23</td>
<td>1.34±0.31</td>
</tr>
<tr>
<td>UKQCD [41]</td>
<td>0.70±0.07</td>
<td>0.66±0.15</td>
<td>1.01±0.13</td>
</tr>
<tr>
<td>ELC [42]</td>
<td>0.64±0.16</td>
<td>0.41±0.28</td>
<td>0.86±0.24</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Group</th>
<th>$A_1(q_{\text{max}}^2)$</th>
<th>$A_2(q_{\text{max}}^2)$</th>
<th>$V(q_{\text{max}}^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modified E791</td>
<td>0.80 ± 0.05</td>
<td>0.56 ± 0.09</td>
<td>1.58 ± 0.14</td>
</tr>
<tr>
<td>ISGW2 [35]</td>
<td>0.70</td>
<td>0.94</td>
<td>1.52</td>
</tr>
</tbody>
</table>

7.3 Test for Quark Models

Unlike $B$ to $D$ semileptonic decays, $D^+ \rightarrow \bar{K}^*0 l^+ \nu_l$ is a semileptonic decay from a heavy quark to a light quark. As a result, HQET is not applicable in a straightforward way and to predict theoretically the branching fraction of this mode is a challenge. The validity of a quark model has been frequently confirmed by the comparison of the predicted branching fraction of such a model with the experimentally measured value. For instance, the ISGW2 model predicts [35]

$$\frac{\Gamma(D \rightarrow K^* e^+ \nu_e)}{\Gamma(D \rightarrow K^0 e^+ \nu_e)} = 0.54.$$  \hspace{1cm} (7.1)

Using the PDG average $\mathcal{B}(D^+ \rightarrow K^0 e^+ \nu_e) = (6.8 \pm 0.8)\%$, [11] the results of this work gives

$$\frac{\Gamma(D \rightarrow K^* e^+ \nu_e)}{\Gamma(D \rightarrow K^0 e^+ \nu_e)} = 0.99 \pm 0.06(\text{stat.}) \pm 0.07(\text{sys.}) \pm 0.06,$$  \hspace{1cm} (7.2)

where the third error is due to the error of $\mathcal{B}(D^+ \rightarrow K^0 e^+ \nu_e)$. Thus, the ISGW2 model prediction is about $4\sigma$ smaller than the value obtained using the results of this work. To provide more useful information for quark models, I need a measurement of $\mathcal{B}(D^+ \rightarrow K^0 e^+ \nu_e)$ with the same data samples used in this work. A measurement
of the ratio of the two branching fractions with a larger data sample would be even more desirable.
APPENDIX A

COMBINING MEASUREMENTS AND ERRORS

The method used to combine measurements and errors in this analysis is well described in one of Paul Avery’s internal CLEO notes. [43]

For two measurements $R_1$ and $R_2$, the combined measurement, $R$, is given by

$$R = \sum_i w_i R_i$$  \hspace{1cm} (A.1)

where $w_i$ is the weight of $R_i$ and given by

$$w_i = \frac{\sum_k V_{ik}^{-1}}{\sum_{jk} V_{jk}^{-1}}.$$  \hspace{1cm} (A.2)

$V$ is the covariance matrix of the measurement $R$ obtained by

$$V = V^{st} + V_1^{sys} + V_2^{sys} + V_3^{sys} + \ldots$$  \hspace{1cm} (A.3)

where $V^{st}$ is the covariant matrix for the statistical error of $R$ and $V_k^{sys}$ is the matrix for the systematic error source $k$. Then, the combined statistical error and each of the systematic errors for the source $k$ are given by

$$\sigma_{st}^2 = \sum_{ij} w_i w_j (V^{st})_{ij}$$  \hspace{1cm} (A.4)

and

$$(\sigma_{sys})_k = \sum_{ij} w_i w_j (V_k^{sys})_{ij}.$$  \hspace{1cm} (A.5)

After figuring out $(\sigma_{sys})_k$’s, the total combined systematic error was obtained by the quadratic sum of $(\sigma_{sys})_k$’s.
When $V_{K^*}^{sy}$ is evaluated, the correlation in the errors among different measurements must be carefully estimated. When the electron (muon) mode of CLEO II.V and CLEO II were combined, the systematic errors from the sources of signal MC model dependence, feed down from $\bar{K}_2^*$, the $D^+ \to \bar{K}^{*0}\pi^0 l^+ \nu_l$ mode contribution and the uncertainty in the $\delta m$ background shape in the normalization mode were assumed to be 100% positively correlated. Since other systematic errors were estimated separately for CLEO II.V and CLEO II, they were treated as uncorrelated errors. For combining the electron and muon modes, the systematic errors from the data-MC difference, signal MC model dependence, efficiency variation due to fragmentation, feed down from the $\bar{K}_2^*$, the $D^+ \to \bar{K}^{*0}\pi^0 l^+ \nu_l$ mode contribution and the uncertainties in the normalization mode $\delta m$ signal and background shapes were assumed to be 100% positively correlated. Again, other systematic errors were considered uncorrelated, having been estimated separately in the electron and the muon modes. Note that assuming 100% positive correlation overestimates the combined error.
APPENDIX B

METHODS OF NEUTRINO MOMENTUM SELECTION

In my standard analysis, the $\nu$ momentum estimate which gives the value of $\delta m$ closest to the $D^*-D$ mass difference was selected. However, one of the disadvantages of this $\nu$ momentum selection method is that background events make a peak at the same place in $\delta m$ as signal events do. To avoid this disadvantage, two other methods to select $\nu$ momentum were tested with a help of generator-level information in MC events.

The first method is to reconstruct the most probable $\nu$ momentum as a function of other reconstructed quantities such as the angle between the thrust axis and the $\nu$ momentum, the angle between the directions of the $\nu$ and the $K\pi l$ combination, the mass of the $K\pi l$ combination, and so on. The multi-dimensional space consisting of the angles and the mass of the $K\pi l$ combination was divided into small cells. The probability of each cell was estimated using a large sample of signal MC (approximately one million events). After counting the number of MC events identified as signal by using the QQ tagging method in each cell, those numbers were properly normalized. The normalized number of signal MC events in each cell was used as the probability of the cell. Using this probability distribution in the multi-dimensional space, the $\nu$ momentum is obtained in two ways: by finding the cell which possesses the highest probability (i) among all of the cells and (ii) among the cells satisfying the $D$ mass constraint. However, according to the study using generic continuum MC events, the reconstructed quantities did not provide enough constraints so that the resolutions of the $\delta m$ peaks in the above two methods were not smaller than that in the standard analysis. In the analyses with real data samples, the resolutions of the $\delta m$ peaks obtained using these two methods were comparable with that of the pseudo $\delta m$ peak. This implies that $\nu$ momentum selection using the probability distribution function determined by other reconstructed quantities does not provide
advantages compared to the analysis using pseudo $\delta m$. Since the standard analysis method gives smaller statistical errors than the pseudo $\delta m$ analysis method does, I decided not to use the method using the probability distribution as an alternative standard analysis method.

The second method also used about one million signal MC events. However, the information obtained from the signal MC events was used to select the $\nu$ momentum among the three $\nu$ momentum estimates, not to reconstruct $\nu$ momentum as in the first method. Using QQ information in the signal MC events, the likelihood of each of the three $\nu$ momentum estimates was calculated and the $\nu$ momentum estimate which had the maximum likelihood was selected as the $\nu$ momentum. The most important thing in the selection of the $\nu$ momentum is how to choose the $\nu$ momentum between the two $\nu$ momentum estimates obtained with the thrust axis. This is because these two $\nu$ momentum estimates take about 90% of the selected $\nu$ momenta with the nominal $\delta m$ selection method (see Fig. 3.3). The major difference between these two estimates is that $\bar{p}_{\nu 1}$ in Fig. 3.1 has smaller angle with $\bar{p}_{K\pi l}$ than $\bar{p}_{\nu 2}$ does. Wrong solution selection affects has a bigger impact on the $\delta m$ resolution when the angle between $\bar{p}_{K\pi l}$ and $\bar{p}_{D^+}$ is small. In this case, $\bar{p}_{\nu 1}$ in Fig. 3.1 makes a small angle with $\bar{p}_{K\pi l}$ and $\bar{p}_{\nu 2}$ makes a large angle. Therefore, in this $\nu$ momentum selection method, the main objective is to select the right $\nu$ momentum when $\bar{p}_{K\pi l}$ and $\bar{p}_{D^+}$ make a small angle. In other words, how to select $\bar{p}_{\nu 2}$ when $\bar{p}_{\nu 2}$ is the right $\bar{p}_{\nu}$. To address this point, the correlations between the $D$ decay configuration in the $D^+$ rest frame and the measured quantities in the lab frame were investigated using generator-level information in signal MC events. It is easier to classify $D$ decay configurations in the $D^+$ rest frame since the angle between $\bar{p}_\nu$ and $\bar{p}_{D^+}$ in the lab frame is smaller than that between $\bar{p}_\nu$ in the $D^+$ rest frame and the direction of $D^+$ measured in the lab frame due to the boost in the lab frame. In the reconstruction process, however, the $D$ decay configuration in the $D^+$ rest frame cannot be determined because $\bar{p}_{D^+}$ is not known. If the correlations between the $D$ decay configuration and the measured quantities in the lab frame are known, then they are helpful in the $\nu$ momentum selection. To investigate these correlations, $D$ decay configurations were separated into seven classes. Six classes are shown in Fig. B.1. Events not belonging to any
of these six classes were classified into the seventh class. For each of these event shapes in the $D^+$ rest frame, histograms of various measured quantities in the lab frame were plotted. Examples are shown in Fig. B.2, Fig. B.3 and Fig. B.4. Among the D decay configuration in Fig. B.1, the event types 1 and 3 will have larger angle between $\vec{p}_{K\pi l}$ and $\vec{p}_\nu$ in the lab frame than other event types. Since the angle between $\vec{p}_l$ and $\vec{p}_{K\pi l}$ in Fig. B.3 is a good quantity to select the event types 1 and 3 in the $D^+$ rest frame, this variable was used to assign the values of likelihoods to the three $\nu$ momentum estimates. Another variable, $|p_\nu|/|p_{K\pi l}|$ in Fig. B.2, was tested to be included in the likelihood estimation with the angle between $\vec{p}_l$ and $\vec{p}_{K\pi l}$, but this had no impact on the $\delta m$ resolution. The likelihood of a $\vec{p}_\nu$ estimate is given by

$$
\mathcal{L}(\vec{p}_\nu) = \sum_i w_i P_i(\angle(\vec{p}_l, \vec{p}_{K\pi l})) P_i(\angle(\vec{p}_\nu, \vec{p}_{K\pi l}))
$$

(B.1)

where the index $i$ runs over the $D$ decay configuration. Here $w_i$ is the weight of the $D$ decay configuration $i$. The number of MC signal events in each of the $D$ decay
Figure B.2: $\frac{|p_\ell|}{p_{K\pi\ell}}$ distributions in the lab frame from the $D$ decay configuration type 1 (top) to the $D$ decay configuration type 7 (bottom).
Figure B.3: distributions of angle between $\vec{p}_l$ and $\vec{p}_{K\pi l}$ in the lab frame from the $D$ decay configuration type 1 (top) to the $D$ decay configuration type 7 (bottom).
Figure B.4: Distributions of angle between $\vec{p}_\nu$ and $\vec{p}_{K\pi l}$ in the lab frame from the $D$ decay configuration type 1 (top) to the $D$ decay configuration type 7 (bottom).
Table B.1: Comparison of $S/\sqrt{N}$'s normalized with that of pseudo $\delta m$.

<table>
<thead>
<tr>
<th>selection with $\delta m$</th>
<th>1.15</th>
</tr>
</thead>
<tbody>
<tr>
<td>with $D$ decay configuration</td>
<td>1.13</td>
</tr>
<tr>
<td>solution 1 in Fig. 3.1</td>
<td>1.11</td>
</tr>
</tbody>
</table>

configurations after applying the analysis cuts was counted and normalized. These normalized numbers of MC signal events were used as the $D$ decay configuration weights. $P_i(\angle(\vec{p}_l, \vec{p}_{K\pi l}))$ in the equation B.1 is the probability for the angle between $\vec{p}_l$ and $\vec{p}_{K\pi l}$ to be $\angle(\vec{p}_l, \vec{p}_{K\pi l})$ in an event-type of type $i$. $P_i(\angle(\vec{p}_\nu, \vec{p}_{K\pi l}))$ in the equation B.1 is the same as $P_i(\angle(\vec{p}_l, \vec{p}_{K\pi l}))$ except using $\vec{p}_\nu$ instead of $\vec{p}_l$. Fig. B.5 shows the two dimensional histogram of $\angle(\vec{p}_l, \vec{p}_{K\pi l})$ versus $\angle(\vec{p}_\nu, \vec{p}_{K\pi l})$ in each of the event types in the lab frame. These two dimensional histograms indicate that the correlations between $\angle(\vec{p}_l, \vec{p}_{K\pi l})$ and $\angle(\vec{p}_\nu, \vec{p}_{K\pi l})$ in the lab frame are small and these correlations were neglected in the likelihood estimation in the equation B.1.

The resultant $\delta m$ resolution with this method is poorer than that with the $\delta m$ selection method and comparable with the $\delta m$ resolution when only the $\vec{p}_{\nu 1}$ in Fig. 3.1 was selected. The $\delta m$ histograms made with signal $D^+ \rightarrow \bar{K}^*0e^+\nu_e$ MC events are shown in Fig. B.6. To make these histograms, the normal analysis cuts were applied but no $K\pi$ mass fits were used. When the $D$ decay configurations were used to select $\vec{p}_\nu$, the resulting distribution of types of $\vec{p}_D$'s is given in Fig. B.7. $S/\sqrt{N}$'s of these three methods were comparable and 11% to 15% higher than that of pseudo $\delta m$ analysis. This result is summarized in Table B.1.

Based on the comparison of $S/\sqrt{N}$'s of various methods of $\vec{p}_\nu$ selection, the selection method with the $D$ decay configuration does not give much gain, and the analysis using this selection method was not pursued further. Instead, the analysis using only $\vec{p}_{\nu 1}$ was carried out and the results were compared with those of normal analysis (see chapter 6).
Figure B.5: Two dimensional histograms of the angle between $\vec{p}_l$ and $\vec{p}_{K\pi l}$ versus angle between $\vec{p}_\nu$ and $\vec{p}_{K\pi l}$ in each of the event types in the lab frame.
Figure B.6: Comparison of $\delta m$ resolutions with various $\nu$ momentum selection methods. These distributions were obtained with signal $D^+ \rightarrow \bar{K}^{*0} e^+ \nu_e$ MC events.

Figure B.7: The distribution of types of $\vec{p}_D$'s when the $\vec{p}_\nu$ selection method with the event types in the $D^+$ rest frame was used.
APPENDIX C
CHECK OF THE NORMALIZATION MODE ANALYSIS

The method of $M_{D^+}$ and $\delta m$ fits in the $D^+ \to K^-\pi^+\pi^+$ analysis was checked using different signal shapes in the $\delta m$ fits. The $\delta m$ plot was obtained by the $D^+$ mass fits in each of $\delta m$ bins as described in chapter 4. Three $\delta m$ fits were carried out using this $\delta m$ plot. The first fit used the signal $\delta m$ shape obtained from QQ tagged combinations from a signal MC sample as described in chapter 4. The second fit used the signal $\delta m$ distribution obtained from a QQ tagged $D^+$ and a “single QQ tagged” slow $\pi^0$. The phrase “single QQ tagged” $\pi^0$ refers to a $\pi^0$ that has at least one daughter identified as a daughter of the signal $\pi^0$ by QQ tagging. The third fit used a $\delta m$ distribution from the signal MC sample made with $D^+$ mass fits without using QQ tagging. Since no QQ tagging method was used in the third category, the signal $\delta m$ shape has the largest tail among the three methods. In all of these three categories, the backgrounds were fit with the function in equation 4.1 with floating parameters $Q_{had}$ and $B_{had}$. As mentioned in chapter 4, the appropriateness of using this $\delta m$ background function was tested with $\pi^0$ sideband data. The comparison of the efficiency-corrected yields in CLEO II.V and CLEO II data measured with these three different $\delta m$ fit methods is given in Table C.1. These three $\delta m$ fit methods give consistent results.

The analysis method using the $M_{D^+}$ cut was also considered. A $\delta m$ plot was made using a $D^+$ mass cut of $1.84$ GeV $\leq m_{D^+} \leq 1.90$ GeV. In the fit of this $\delta m$ plot to

Table C.1: The efficiency-corrected yields in CLEO II.V and CLEO II data with different $\delta m$ fits in the normalization mode analysis.

<table>
<thead>
<tr>
<th></th>
<th>CLEO II.V</th>
<th>CLEO II</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{D^+}$ fits + QQ tagged sig.</td>
<td>$158015.0 \pm 2503.3$</td>
<td>$74494.5 \pm 1723.0$</td>
</tr>
<tr>
<td>$M_{D^+}$ fits + single QQ $\pi^0$ sig.</td>
<td>$157797.7 \pm 2512.5$</td>
<td></td>
</tr>
<tr>
<td>$M_{D^+}$ fits + non-QQ tagged sig.</td>
<td>$159846.7 \pm 2544.6$</td>
<td>$77555.0 \pm 1798.6$</td>
</tr>
</tbody>
</table>
obtain the number of normalization mode events, three methods to determine the background function were studied. The first was to use the background function in equation 4.1. In the second method, a $\delta m$ distribution obtained with the data events in $D^+$ mass sidebands was used. The $D^+$ mass sidebands were defined by $1.750 \text{ GeV} \leq M_{D^+} \leq 1.810 \text{ GeV}$ for the low and $1.925 \text{ GeV} \leq M_{D^+} \leq 1.985 \text{ GeV}$ for the high sideband. The third method used a continuum MC sample. By using the QQ tagging technique, only background events in the $D^+$ mass signal band were selected. Then, the $\delta m$ plot was made with these background events and used as the background function in the $\delta m$ fit. The efficiency-corrected yields of CLEO II.V data measured with these three different $\delta m$ background shapes are shown in Table C.2. As can be seen in Table C.2, the efficiency-corrected yields with these different methods are not consistent. For reference, the $\delta m$ fits with was these different $\delta m$ background shapes used the same signal shape obtained with a signal MC sample using the same $M_{D^+}$ cut.

In the first method using the background function in equation 4.1, a question is whether or not the $\delta m$ background function in equation 4.1 is appropriate to be used as the $\delta m$ background fit function. To answer this question, a $\delta m$ fit was executed while floating all of the $\delta m$ background function’s parameters for the $M_{D^+}$ signal band data. Then, the $\delta m$ background shape after the fit was compared with the shape of the $\delta m$ distribution of the data $M_{D^+}$ sidebands and that of the background continuum MC events in the $M_{D^+}$ signal band. Fig. C.1 shows that the background shape with the background function in equation 4.1 is very different from the $\delta m$ shape of the data $M_{D^+}$ sidebands and also from that of the background continuum MC events in the $M_{D^+}$ signal band especially where the $\delta m$ signal peak is located. This comparison shows that using the background function in equation 4.1 is not
appropriate in the $\delta m$ fit when the $M_{D^+}$ cut is used.

Using the $\delta m$ distribution from the $M_{D^+}$ sidebands data as the $\delta m$ background fit function, there remains a question whether or not the $M_{D^+}$ sidebands give the same $\delta m$ shape as that of the background in the $M_{D^+}$ signal band. This question was studied with a continuum MC sample. After making a $\delta m$ plot with the $M_{D^+}$ sidebands events, the shape of this $\delta m$ plot was compared with that of a $\delta m$ distribution obtained with background events in the $M_{D^+}$ signal band. This comparison is shown in Fig. C.2. In the first seven bins, which correspond to the first 14 bins in the $D^+ \to K^-\pi^+\pi^+$ analysis $\delta m$ plot since the bin size in Fig. C.2 is double the normal, the $M_{D^+}$ signal band background events have more entries than the $M_{D^+}$ sideband events do. Since these first $\delta m$ bins are the most important in extracting the number of signal events from a $\delta m$ fit, using the $\delta m$ distribution of the data $M_{D^+}$ sidebands as the $\delta m$ background fit function is questionable.

The last method, in which the $\delta m$ shape made with continuum MC background events in the $M_{D^+}$ signal band, works only if a continuum MC sample is consistent with data. To check this, $\delta m$ plots of a continuum MC sample and CLEO II.V data were made and compared in the low and the high $M_{D^+}$ side band separately.

Figure C.1: Comparison of $\delta m$ background shapes.
These comparisons are given in Fig. C.3. These comparisons reveal that the $M_{D^+}$ low sideband $\delta m$ shapes agree well between the continuum MC sample and data but the high sideband $\delta m$ shapes disagree in the first three bins (first 6 bins in the normal analysis $\delta m$ plots) systematically. Since these three $\delta m$ bins play a crucial role in $\delta m$ fits, the consistency of continuum MC to data is questionable. As a result, it would not be reliable to use the $\delta m$ shape made from background events in a continuum MC sample in the $M_{D^+}$ signal band as the background fit function in a $\delta m$ fit.

Based on these reasons, the $D^+ \rightarrow K^-\pi^+\pi^+$ analysis with the $M_{D^+}$ cut was not included in $R_t$ measurement.
Figure C.3: Comparisons of $\delta m$ shapes of a continuum MC sample with those of data in the $M_{D^+}$ low sideband (left) and the high sideband (right).
APPENDIX D

ESTIMATION OF SYSTEMATIC ERROR IN ELECTRON ID EFFICIENCY

To estimate the systematic error in electron identification efficiency, electrons from radiative-bhabha events were embedded into hadron data samples. Since I know the number of electrons embedded, I can calculate the electron identification efficiency by counting the number of electrons identified in the radiative-bhabha embedded hadron data samples. Unlike electrons in $B\bar{B}$ events, the direction of an electron in a continuum events are correlated with the jet direction. If this effect is not taken into account, this embedding study may result in misleading conclusion. A “weighting” method was used to deal with this issue as described below. In this note, the following steps will be described in detail:

1. selection criteria of radiative bhabha events,

2. determination of weighting factors to make the event shape of the radiative-bhabha embedded hadron events be similar to that of events in $D^+ \rightarrow \bar{K}^*{0}e^+\nu_e$ analysis,

3. electron identification efficiency in $D^+ \rightarrow \bar{K}^*{0}e^+\nu_e$ analysis, and

4. systematic error sources in the electron identification efficiency and the process of the systematic error estimation.

D.1 Selection of Radiative Bhabha Events

The CLEO electron identification working group [44] made criteria to select radiative bhabha events as following:

- two good oppositely charged tracks,
• sum of cluster energies $> 9.0\text{GeV}$,
• $\text{Max} \left( \cos \theta \text{ between two clusters } \right) < 0.9$,
• $\cos \theta \text{ between two tracks } > -0.95$, and
• $0.4\text{GeV} < \text{Momentum of both track } < 6.0\text{GeV}$.

Furthermore, there are radiative bhabha event selection criteria in the CLEO event classification routine given by [45]:

• number of good tracks in the DR is either one or two,
• at least three showers but no more than eight,
• the CC deposited energy of the third most energetic shower $> 300\text{MeV}$,
• total energy deposited in the CC $> 6.0\text{GeV}$,
• at least two of the three most energetic showers in the barrel CC,
• the CC deposited energy of the fourth shower $< 150\text{MeV}$, and
• $\cos \theta \text{ between two showers } < 0.95$.

### D.2 Event Shape Weighting Factors

Since $B\bar{B}$ events are used as hadron data for the radiative-bhabha embedding study at CLEO, the event shape of these hadron data is different from that of events in $D^+ \rightarrow \bar{K}^{*0}e^+\nu_e$ analysis. Furthermore, the embedded electrons are not kinematically correlated with the particles of the hadron events if the $\theta$ dependence of the differential cross sections of radiative bhabha events and $q\bar{q}$ events is ignored. To make the event shape of the radiative-bhabha embedded hadron events similar to that of $D^+ \rightarrow \bar{K}^{*0}e^+\nu_e$ analysis events, and to give kinematic correlation between the embedded electrons and the other particles of hadron events, two variables, the thrust of an event and the cosine of the angle between the electron momentum and the thrust axis, were used. By weighting each radiative-bhabha embedded hadron events based on the value
Figure D.1: Two dimensional scattered plot of the thrust and the cosine of the angle between the electron momentum and the thrust axis. This plot was made with a $D^+ \rightarrow \bar{K}^{*0} e^+ \nu_e$ signal MC sample using the analysis cuts in Table 3.1.

of these two parameters, the two-dimensional distribution of these two parameters of the embedded sample was made similar to that of data events in $D^+ \rightarrow \bar{K}^{*0} e^+ \nu_e$ analysis.

To consider the correlation between the thrust and the cosine, shown in Fig. D.1 and Fig. D.2, a three-dimensional weighting process was adopted. Since the electron identification efficiency is measured bin by bin of electron momentum, a two-dimensional weighting process was executed for each of the electron momentum bins. The two-dimensional plot in Fig. D.3 shows the space spanned by the thrust and the cosine of the angle between the electron momentum and the thrust axis. The electron identification efficiency $\varepsilon_i$ of the $i$th bin of electron momentum is defined by

$$
\varepsilon_i = \frac{N'_i}{N_i},
$$

(D.1)

where $N_i$ and $N'_i$ are number of embedded electrons and number of identified electrons in the $i$th bin of $|\vec{p}_e|$. When $(n)_j k$ and $(n')_j k$ are defined by the number of embedded electrons and number of identified electrons in the $(j, k)$ bin in thrust-cosine space,
Figure D.2: Two dimensional scattered plots between electron momentum and thrust and between electron momentum (left) and cosine of the angle between electron momentum and thrust axis (right). These plots were made with a $D^+ \rightarrow \bar{K}^*0\pi^+\nu_e$ signal MC sample using the analysis cuts in Table 3.1.

For $i$ th bin of electron momentum:

$$\text{bin}(i, j, k)$$

$k$ th bin of $\cos(t, p_e)$

Figure D.3: The space spanned by the thrust and the cosine of the angle between the electron momentum and the thrust axis.
then equation D.1 can be rewritten as
\[
\epsilon_i = \frac{\sum_{j,k} (n'_{ij})_{jk}}{\sum_{j,k} (n_{ij})_{jk}}. 
\]  
(D.2)

However, this is not a good estimate of electron identification efficiency if \( \frac{(n'_{ij})_{jk}}{(n_{ij})_{jk}} \) is not uniform in thrust-cosine space and if \((n_{ij})_{jk} \) distribution is not close to that of \( D^+ \rightarrow \bar{K}^*0e^+\nu_e \) signal. To avoid this potential problem, I introduced a weighting method to make the \((n_{ij})_{jk} \) distribution the same as that of signal MC. The weighted electron identification efficiency in the \( i \)th bin of |\( p_T \)|, \( \epsilon_{wi} \), is defined by
\[
\epsilon_{wi} = \frac{N'_{wi}}{N_{wi}}, 
\]  
(D.3)

where \( N_{wi} \) and \( N'_{wi} \) are weighted number of embedded electrons and weighted number of identified electrons in the bin, respectively. Since \( N_{wi} \) and \( N'_{wi} \) are represented by
\[
N_{wi} = \sum_{j,k} (n_{ij})_{jk} \cdot (w_i)_{jk},
\]  
and
\[
N'_{wi} = \sum_{j,k} (n'_{ij})_{jk} \cdot (w_i)_{jk},
\]  
(D.4)

the weighted electron identification efficiency \( \epsilon_{wi} \) can be expressed as
\[
\epsilon_{wi} = \frac{\sum_{j,k} (n'_{ij})_{jk} \cdot (w_i)_{jk}}{\sum_{j,k} (n_{ij})_{jk} \cdot (w_i)_{jk}}.
\]  
(D.6)

If I use a weight of
\[
(w_i)_{jk} = \frac{(s_i)_{jk}}{(n_{ij})_{jk}},
\]  
(D.7)

where \((s_i)_{jk}\) is the number of electrons in \((k, k)\) bin in a \( D^+ \rightarrow \bar{K}^*0e^+\nu_e \) signal MC, the resultant electron distribution of hadron events is the same as that of \( D^+ \rightarrow \bar{K}^*0e^+\nu_e \) signal MC events.
D.3 Electron ID efficiency in This Analysis

The electron identification efficiency depends on three variables:

- electron decision cut (R2ELEC > 3.0 in this analysis),
- regional dependence on the CC, and
- electron momentum.

In this note, the electron efficiency was measured with R2ELEC > 3.0 cut, on “good-” and “bad-barrel” of the CC separately and over 0.6GeV < |\vec{p}_e| < 2.8GeV.

The CLEO II.V and CLEO II electron identification efficiencies of weighted and unweighted radiative-bhabha embedded hadron data for the CC good- and bad-barrel and are shown in Fig. D.4. Fig. D.5 shows the overlay of the weighted electron identification efficiencies of CLEO II.V and CLEO II CC good-barrel data. The difference between the CLEO II.V and CLEO II electron identification efficiencies in the high electron momentum range can be seen by comparing the un-weighted electron identification efficiencies in Fig. D.4. However, the electron identification efficiency in the high electron momentum range is not important in D⁺ → \bar{K}⁺\pi⁻\nu_e analyses because electrons are soft in D⁺ → \bar{K}⁺\pi⁻\nu_e signal events (see Fig. D.6). Fig. D.6 shows the correspondence between weighted electron identification efficiencies in the CC good-barrel and electron momenta in D⁺ → \bar{K}⁺\pi⁻\nu_e signal MC events.

The weighted electron identification efficiencies measured in this study were used for calculating D⁺ → \bar{K}⁺\pi⁻\nu_e signal MC efficiencies. Using QQ tagging, real electrons in a D⁺ → \bar{K}⁺\pi⁻\nu_e signal MC sample were selected. For these real electrons, the R2ELEC cut was not applied. The R2ELEC cut was applied only for non-electron tracks identified as electrons (fake electrons) in the D⁺ → \bar{K}⁺\pi⁻\nu_e signal MC analysis. In this signal MC analysis, however, the ratio of the number of the fake electrons to that of the real electrons is less than 1.5%. As a result, even if there is any difference in electron fake rate between MC and data, the effect of this difference in the calculation of D⁺ → \bar{K}⁺\pi⁻\nu_e signal MC efficiencies would be negligible. The numbers of combinations of K⁻, π⁺, slow-\pi⁰ and the QQ tagged real electrons were
Figure D.4: Electron identification efficiencies in the CLEO II.V data on the CC good-barrel (top left), bad-barrel (top right) and CLEO II data on the CC good-barrel (bottom left), bad-barrel (bottom right) measured with weighted and un-weighted radiative-bhabha embedded hadron events.
Figure D.5: Comparison of weighted electron identification efficiencies in the CLEO II.V and CLEO II CC good- (left) and bad-barrel (right) data.

Figure D.6: Correspondence between weighted electron identification efficiencies in the CC good-barrel and $|p_{\ell}|$ in $D^+ \rightarrow \bar{K}^{(*)}e^+\nu_e$ signal MC events.
scaled by the weighted electron identification efficiencies of the corresponding electron momentum bins. The efficiency of signal MC analysis was calculated by the ratio of the sum of the scaled real electron combinations and the fake electron combinations divided by the number of generated signal MC events.

**D.4 Estimation of Systematic Error**

The following factors were considered as the sources of systematic error in the electron identification efficiency:

- quality of embedded radiative bhabha events,
- the difference in track and shower matching information of electrons in radiative bhabha events and in analysis hadron events, and
- systematic errors in the weighting factors.

The systematic error source related to the quality of embedded radiative bhabha events was parameterized with three parameters: (i) the angle \( \theta_{\text{emb}} \) between the pre-embedded electron track and the embedded electron track found in a hadron event, (ii) the sum of energy of electron (or positron) that radiated photon and the energy of the radiated photon deposited in the CC, and (iii) \( \chi^2 \) of a radiative bhabha event, which is defined by

\[
\chi^2 = \left( \frac{2.0 \cdot E_{\text{beam}} - E}{\delta E} \right)^2 + \sum_{i=1}^{3} \left( \frac{p_i}{\delta p_i} \right)^2,
\]

(D.8)

where \( E_{\text{beam}} \) is the beam energy and \( E \) and \( p_i \) are the energy and the \( i \)th component of momentum of the electron, positron and photon system. The cut on \( \theta_{\text{emb}} \) made by the electron identification working group to measure the electron identification efficiency is \( \theta_{\text{emb}} < 0.0895 \) radians for good barrel and \( \theta_{\text{emb}} > 0.2003 \) radians for other regions. [46] To estimate the systematic error due to the angle between the pre-embedded electron track and the embedded electron track, the \( \theta_{\text{emb}} \) cut made by the electron identification working group was loosened to \( \theta_{\text{emb}} > 0.2003 \) radians
for good barrel and $\theta_{emb} > 0.2838$ radians for other regions. The variation of the efficiency according to the change of $\theta_{emb}$ cut was assigned as the systematic error for this source.

The electron identification working group required that the sum of the energy of the radiating electron (or positron) and the radiated photon energy deposited in the CC to be less than 3.0GeV. \cite{46} The normalized variation of the electron identification efficiency without this cut was assigned as the systematic error for this systematic error source.

Like the above, the electron identification working group required the $\chi^2$ of a radiative bhabha event to be less than 100.0 in their electron identification efficiency measurement. \cite{46} The systematic error related to $\chi^2$ of a radiative bhabha event was assigned with the efficiency variation by changing this $\chi^2$ cut to $\chi^2 < 10.0$. As shown in Fig. D.7, when $\chi^2$ is less than 10.0, the peak near zero on the distribution of $\chi^2$ probability disappears. Therefore, $\chi^2 < 10.0$ would be a reasonable cut to estimate the systematic error.

The electron identification efficiency variations according to changes on cuts of $\cos \theta_{emb}$, the sum of energies of electron and the radiated photon deposited in the CC, and $\chi^2$ of a radiative bhabha event are summarized in Fig. D.8 and Fig. D.9 for CLEO II. and CLEO II., respectively.

In radiative bhabha events, the matching between the electron or positron track and its cluster is required. As a result, if the electron efficiency is estimated by the ratio of the number of embedded electron or positron tracks found in hadron events with respect to the initial number of embedded electron or positron tracks, the efficiency would be overestimated because the initial electron or positron tracks in radiative bhabha events already passed the matching requirement. To resolve this overestimation problem in the electron identification efficiency, the electron identification working group measured electron identification efficiency using radiative bhabha events selected without the matching requirement. The measured electron identification efficiency with this method was 2.5% lower in $|p_e| < 1.0$GeV and 1.0% lower for other electron momentum range than that with the matching requirement in the radiative bhabha event selection. \cite{46} This difference in the electron identification
Figure D.7: Distributions of radiative-bhabha events $\chi^2$ probability with various $\chi^2$ cuts.
Figure D.8: Electron identification efficiency variations according to changes on various cuts in CLEO II.V radiative-bhabha embedded hadron events.
Figure D.9: Electron identification efficiency variations according to changes on various cuts in CLEO II radiative-bhabha embedded hadron events.
efficiency was assigned as the systematic error for this systematic error source.

The weighting factors related to thrust was obtained using a $D^+ \rightarrow \bar{K}^*0e^+\nu_e$ signal MC sample. A thrust distribution was obtained with a signal MC sample with applying the analysis cuts in Table 3.1. Then, this thrust distribution was fit with a bifurcated-Gaussian function. This fit result is shown in Fig. D.10. The variation of the thrust weighting factors was estimated by the variations of the fit parameters obtained from the fit with the bifurcated-Gaussian function. Fig. D.11 shows the maximum excursion at every point of the thrust fit function with respect to the central values of fit parameters. Three sets of the thrust weighting factors were obtained with these three fit function shapes. The electron identification efficiencies obtained are shown in Fig. D.12. As can be recognized in Fig. D.12, the variation of the electron identification efficiencies due to the variation in the thrust weighting factors is negligible. As a result, the systematic error in the electron identification efficiency due to this variation was ignored.

The systematic error due to the variation of the weighting factors of the cosine of the angle between the electron momentum and the thrust axis of an event was
Figure D.11: Variation of thrust distribution fits according to the variations of fit parameters.

Figure D.12: Comparison of electron identification efficiencies with varying thrust weighting factors for CLEO II.V (left) and CLEO II (right).
Figure D.13: The fit of the cosine distribution with an exponential function function in equation D.9 to determine the cosine weighting factors. The cosine distribution was obtained from a $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$ signal MC sample with the analysis cuts in Table 3.1.

estimated in the same method as in the estimation of the thrust weighting factor systematic error. The only difference was the fit function used to obtain the weighting factor was an exponential function defined by

$$A \times \exp(-B(x - C)),$$

(D.9)

where $B$ and $C$ are fit parameters corresponding to $Slope$ and $Offset$ in Fig. D.13, respectively. The fit result for the cosine weighting factor is shown in Fig. D.13, and the variation of the weighting factors due to the variation of the fit parameters shown in Fig. D.14. The effect on the electron identification efficiencies due to the variation of the weighting factors of the cosine of the angle between electron momentum and thrust axis of an event is given in Fig. D.15. Like in the case of the thrust weighting factor, the variation in the electron identification efficiencies in Fig. D.15 was negligible. As a result, the systematic error due to the variation of the weighting factors of the cosine of the angle between electron momentum and thrust axis of an event is neglected.
Figure D.14: Variation of the cosine distribution fits according to the variations of fit parameters in equation D.9.

Figure D.15: Comparison of electron identification efficiencies with varying the cosine weighting factors for CLEO II.V (left) and CLEO II (right).
Table D.1: Systematic errors in electron identification efficiency in this $D^+ \rightarrow \bar{K}^*0 e^+ \nu_e$ analysis.

<table>
<thead>
<tr>
<th>source</th>
<th>CLEO II.V</th>
<th>CLEO II</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_{emb}$</td>
<td>0.005</td>
<td>0.01</td>
</tr>
<tr>
<td>energy of rad. $e$ and $\gamma$</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>$\chi^2$ of rad. bhabha events</td>
<td>0.005</td>
<td>0.005</td>
</tr>
<tr>
<td>diff. of $e$ in rad. bhabha and had. events</td>
<td>0.015</td>
<td>0.015</td>
</tr>
<tr>
<td>weighting factors</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>0.02</strong></td>
<td><strong>0.02</strong></td>
</tr>
</tbody>
</table>

Since all systematic errors in the electron identification efficiency depend on electron momentum, each systematic error was calculated by a weighted average. Then, total systematic error in the electron identification efficiency was obtained by the quadratic sum of these weighted average systematic errors. The weights were obtained by electron momentum spectrum of a $D^+ \rightarrow \bar{K}^*0 e^+ \nu_e$ signal MC sample with applying the analysis cuts.

As a summary, Table D.1 shows the individual and total systematic errors in the electron identification efficiency in this $D^+ \rightarrow \bar{K}^*0 e^+ \nu_e$ analysis.
APPENDIX E
DIFFERENCE IN $R_E$ MEASUREMENTS BETWEEN CLEO II.V AND CLEO II

The ratio of $R_e$ of CLEO II.V with respect to that of CLEO II is $0.783 \pm 0.121$. Thus, its deviation from 1.0 is about 1.8$\sigma$’s away. To figure out whether this deviation was made by statistical fluctuations or by other potential mistakes in the analysis process, the followings checks were made:

1. Whether the data used in this analysis was correctly selected.

2. Whether the same analysis code was used for both of the CLEO II.V and CLEO II analyses.

3. Determination of what is responsible for the difference in $R_e$’s among raw yields and efficiencies of CLEO II.V and CLEO II $D^+ \rightarrow K^*0e^+\nu_e$ analyses and those in the $D^+ \rightarrow K^-\pi^+\pi^+$ analyses.

4. Comparison of the number of $\pi^0$’s in the CLEO II.V data with that in the CLEO II data found with the $\pi^0$ selection cuts in Table 3.1.

5. Comparison of the numbers of tracks passing the track quality cuts in the CLEO II.V and CLEO II data.

6. Comparison of the numbers of electrons found in the CLEO II.V and CLEO II data with the electron identification cuts in Table 3.1.

7. Comparison of the number of entries in a CLEO II.V inclusive $M_{K\pi}$ plot with that of CLEO II.

8. Comparison of the efficiency-corrected yields obtained with the full and the $\nu$ reconstruction methods in the $D^+ \rightarrow K^-\pi^+\pi^+$ mode for both CLEO II.V
and CLEO II data. This comparison will reveal whether the $\nu$ reconstruction method alters the analysis results.

To check whether the skim process for this analysis was correctly done, I reanalyzed everything with the CLEO official hadron skim and found the final results were the same as those obtained with the skim used in this analysis. With this comparison, it was confirmed that there were no mistakes in the skim process.

In the analysis code, all run number dependence or SVX dependence was removed. In this way it was confirmed that the same analysis code was used for both of the CLEO II.V and CLEO II analyses.

Table E.1 shows the comparison in various CLEO II.V and CLEO II quantities. This comparison elucidates three things:

- CLEO II.V and CLEO II MC samples give the same results.

- The raw yields of CLEO II.V and CLEO II in $D^+ \rightarrow \bar{K}^{*0}\pi^+\nu_\tau$ mode make the CLEO II.V $R_\tau$ be smaller than that of CLEO II by 15.0%, which was estimated by comparing the ratio of the raw yields with the ratio of the total luminosities of CLEO II.V and CLEO II.

- The raw yields of $D^+ \rightarrow K^-\pi^+\pi^+$ mode also make the CLEO II.V $R_\tau$ be smaller by about 5.0% than that of CLEO II.

To solve the puzzle of the difference in the raw yields, the numbers of $\pi_0$'s and tracks in the CLEO II.V and CLEO II data passing the $\pi_0$ selection and the track quality cuts in Table 3.1 were compared. For this study, every twentieth event of CLEO II.V and CLEO II data was used. Table E.2 shows the comparison of the numbers of $\pi_0$'s between CLEO II.V and CLEO II. Fig. E.1 shows the number of $\pi_0$'s comparisons between CLEO II.V and CLEO II $\Upsilon(4S)$ on-, off-resonance and all data.

Table E.3 shows the comparison of the number of tracks between CLEO II.V and CLEO II. The CLEO II.V numbers were scaled with the ratio of the luminosities of CLEO II.V and CLEO II. Fig. E.2 shows the comparisons of the number of tracks between CLEO II.V and CLEO II $\Upsilon(4S)$ on-, off-resonance and all data.
Table E.1: Comparison of CLEO II.V and CLEO II quantities.

<table>
<thead>
<tr>
<th>quantity</th>
<th>CLEO II.V</th>
<th>CLEO II</th>
<th>II.V/II</th>
</tr>
</thead>
<tbody>
<tr>
<td>luminosity (fb⁻¹)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>8.778</td>
<td>4.746</td>
<td>1.850</td>
</tr>
<tr>
<td>Υ(4S) on</td>
<td>5.996</td>
<td>3.137</td>
<td>1.911</td>
</tr>
<tr>
<td>Υ(4S) Off</td>
<td>2.744</td>
<td>1.608</td>
<td>1.706</td>
</tr>
<tr>
<td>D⁺ → K⁺π⁺π⁻ rate</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>raw yield</td>
<td>3723.8 ± 283.2</td>
<td>2315.7 ± 193.7</td>
<td>1.608 ± 0.182</td>
</tr>
<tr>
<td>efficiency</td>
<td>0.050 ± 0.001</td>
<td>0.050 ± 0.001</td>
<td>1.000 ± 0.020</td>
</tr>
<tr>
<td>D⁺ → K⁻π⁺π⁺ mode</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M_D fits and δm fit</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>raw yield</td>
<td>10018.0 ± 158.7</td>
<td>5169.1 ± 119.6</td>
<td>1.938 ± 0.054</td>
</tr>
<tr>
<td>efficiency</td>
<td>0.063 ± 0.001</td>
<td>0.063 ± 0.001</td>
<td>1.000 ± 0.005</td>
</tr>
<tr>
<td>δm fit sideband supt.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>raw yield</td>
<td>10240.1 ± 181.7</td>
<td>5339.4 ± 142.9</td>
<td>1.918 ± 0.062</td>
</tr>
<tr>
<td>efficiency</td>
<td>0.063 ± 0.001</td>
<td>0.065 ± 0.001</td>
<td>0.960 ± 0.011</td>
</tr>
</tbody>
</table>

Table E.2: Comparison of the number of π⁰'s in the CLEO II.V data with that of CLEO II. CLEO II numbers were scaled by the ratio of the corresponding luminosities of CLEO II with respect to those of CLEO II.V. The number of events in the table means the number of events which have at least one π⁰ found with the selection cuts in Table 3.1. The average numbers of π⁰'s per event were calculated among those events.

<table>
<thead>
<tr>
<th></th>
<th>CLEO II.V</th>
<th>CLEO II</th>
<th>II.V/II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Υ(4S) On and Off</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>number of events</td>
<td>1856486.0 ± 1362.53</td>
<td>1790919.2 ± 1820.22</td>
<td>1.037 ± 0.001</td>
</tr>
<tr>
<td>average no. of π⁰'s per event</td>
<td>5.740 ± 0.004</td>
<td>5.711 ± 0.006</td>
<td>1.005 ± 0.001</td>
</tr>
<tr>
<td>total number of π⁰'s</td>
<td>10656229.0 ± 10784.78</td>
<td>10227940.0 ± 14950.85</td>
<td>1.042 ± 0.002</td>
</tr>
<tr>
<td>Υ(4S) On</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>number of events</td>
<td>1371740.0 ± 1171.21</td>
<td>1332063.1 ± 1595.49</td>
<td>1.030 ± 0.002</td>
</tr>
<tr>
<td>average no. of π⁰'s per event</td>
<td>5.958 ± 0.005</td>
<td>5.968 ± 0.007</td>
<td>0.998 ± 0.001</td>
</tr>
<tr>
<td>total number of π⁰'s</td>
<td>8172827.0 ± 9784.438</td>
<td>7949752.0 ± 13327.10</td>
<td>1.028 ± 0.002</td>
</tr>
<tr>
<td>Υ(4S) Off</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>number of events</td>
<td>484746.0 ± 696.237</td>
<td>470277.8 ± 895.709</td>
<td>1.031 ± 0.002</td>
</tr>
<tr>
<td>average no. of π⁰'s per event</td>
<td>5.124 ± 0.007</td>
<td>5.110 ± 0.010</td>
<td>1.003 ± 0.002</td>
</tr>
<tr>
<td>total number of π⁰'s</td>
<td>2483838.0 ± 4923.530</td>
<td>2403120.0 ± 6562.448</td>
<td>1.034 ± 0.003</td>
</tr>
</tbody>
</table>
Figure E.1: Comparisons of the numbers of $\pi^0$s between CLEO II.V and CLEO II $\Upsilon(4S)$ on- (left), off-resonance (middle) and all data (right). CLEO II.V plots were scaled with luminosities.

Table E.3: Comparison of number of the tracks between CLEO II.V and CLEO II. CLEO II.V numbers were scaled with the ratio of the corresponding luminosities of CLEO II.V with respect to those of CLEO II.

<table>
<thead>
<tr>
<th></th>
<th>CLEO II.V</th>
<th>CLEO II</th>
<th>II.V/ II</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Upsilon(4S)$ On and Off</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>number of events</td>
<td>528840.0 ± 534.737</td>
<td>521529.0 ± 722.170</td>
<td>1.014 ± 0.002</td>
</tr>
<tr>
<td>average no. of tracks per event</td>
<td>7.960 ± 0.008</td>
<td>7.780 ± 0.011</td>
<td>1.021 ± 0.002</td>
</tr>
<tr>
<td>total number of tracks</td>
<td>4209566.0 ± 6001.403</td>
<td>4057496.0 ± 8029.846</td>
<td>1.037 ± 0.003</td>
</tr>
<tr>
<td>$\Upsilon(4S)$ On</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>number of events</td>
<td>369817.0 ± 439.839</td>
<td>364698.0 ± 603.902</td>
<td>1.014 ± 0.002</td>
</tr>
<tr>
<td>average no. of tracks per event</td>
<td>8.147 ± 0.010</td>
<td>8.001 ± 0.013</td>
<td>1.018 ± 0.002</td>
</tr>
<tr>
<td>total number of tracks</td>
<td>3012899.0 ± 5149.465</td>
<td>2917949.0 ± 6769.362</td>
<td>1.033 ± 0.003</td>
</tr>
<tr>
<td>$\Upsilon(4S)$ Off</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>number of events</td>
<td>158894.0 ± 305.159</td>
<td>156831.0 ± 396.019</td>
<td>1.013 ± 0.003</td>
</tr>
<tr>
<td>average no. of tracks per event</td>
<td>7.472 ± 0.014</td>
<td>7.332 ± 0.019</td>
<td>1.019 ± 0.003</td>
</tr>
<tr>
<td>total number of tracks</td>
<td>1187256.0 ± 3185.521</td>
<td>1149885.0 ± 4160.541</td>
<td>1.033 ± 0.005</td>
</tr>
</tbody>
</table>
Figure E.2: Comparisons of the number of tracks between CLEO II.V and CLEO II Y(4S) on- (left), off-resonance (middle) and all data (right).

The numbers of electrons in the CLEO II.V and CLEO II data identified with the electron identification cuts in Table 3.1 were also compared. Fig. E.3 shows the comparison of the electron momentum spectrum of CLEO II.V and CLEO II in $B\bar{B}$ events. To obtain the electron momentum spectrum in $B\bar{B}$ events, continuum subtraction was done data set by data set to minimize experimental environment effects. The CLEO II.V electron momentum spectrum was scaled by the ratio of the CLEO II and CLEO II.V total luminosities. The numbers of electrons found are 57096.7 ± 238.949 in CLEO II.V data and 30721.9 ± 175.277 in CLEO II. The non-integer numbers of electrons are due to the continuum subtraction. If the number of electrons in CLEO II.V were scaled with the ratio of the total luminosities of CLEO II and CLEO II.V, the scaled number of electrons in CLEO II.V will be 30863.1 ± 128.866. Then, the ratio of the scaled number of electrons in CLEO II.V with respect to the number of CLEO II electrons would be 1.005 ± 0.007. Therefore, the number of electrons found with the electron identification cuts in Table 3.1 agrees well between CLEO II.V and CLEO II. As a result, the difference in the number of electrons between CLEO II.V and CLEO II had a negligible impact on the study of the difference in $R_e$’s of CLEO II.V and CLEO II.

In the CLEO II.V and CLEO II inclusive $M_{K\pi}$ plots, if the CLEO II.V plot was scaled with the ratio of CLEO II and CLEO II.V luminosities, the ratio of the numbers of entries would be the square of the ratio of “total number of tracks” in Table E.3.
Figure E.3: Comparison of the electron momentum spectrum of CLEO II.V and CLEO II in $B\bar{B}$ events.

To make an inclusive $M_{K\pi}$ plot, all possible combinations of tracks passing the track quality cuts in Table 3.1 were considered. In each of the combinations, one track was reconstructed with the $\pi$ hypothesis and the other with the $K$ hypothesis. The expected ratios of entries of CLEO II.V (scaled) and CLEO II inclusive $M_{K\pi}$ plots are $1.067 \pm 0.006$, $1.067 \pm 0.010$ and $1.075 \pm 0.006$ for $\Upsilon(4S)$ on-, off-resonance and all data. The ratios of entries are $1.043 \pm 0.001$, $1.042 \pm 0.001$ and $1.045 \pm 0.001$ for $\Upsilon(4S)$ on-, off-resonance and all data, respectively. These ratios are slightly less than those expected from the comparison of numbers of tracks in CLEO II.V and CLEO II. Fig. E.4 shows the CLEO II.V (scaled) and CLEO II inclusive $M_{K\pi}$ plots for $\Upsilon(4S)$ on-, off-resonance and all data.

To decide whether or not the raw yield difference between CLEO II.V and CLEO II $D^+ \rightarrow K^-\pi^+\pi^+$ modes was made by statistical fluctuations, the amount of background in the CLEO II.V (scaled with luminosity) $M_{K^-\pi^+\pi^+}$ plot was compared with that of CLEO II because the results of the comparison of the number of $\pi^0$s and tracks could be used for the background rather than for the signal. From the comparison of the numbers of $\pi^0$s and tracks in CLEO II.V and CLEO II, the ratio of the amount of background in $M_{K^-\pi^+\pi^+}$ plots is expected to be $1.115 \pm 0.009$ which
Figure E.4: CLEO II.V (scaled) and CLEO II inclusive $M_{K\pi}$ plots for $\Upsilon(4S)$ on-(left), off-resonance (middle) and all data (right).

...is the cube of the ratio of the "total number of tracks" entry in Table E.3. If the effect of the difference in the number of $\pi^0$'s between CLEO II.V and CLEO II is considered, the ratio of the backgrounds can reach up to $1.162 \pm 0.010$. The reason to consider the effect of the difference in number of $\pi^0$'s is that the $M_{K^-\pi^+\pi^+}$ plots were made with ntuples composed of all possible $K^-$, $\pi^+$, $\pi^+$ and $\pi^0$ combinations. To compare the background amount in $M_{K^-\pi^+\pi^+}$ plots, the area of the sidebands in the plots can be used. However, I just compared the whole areas of $M_{K^-\pi^+\pi^+}$ plots in the $\delta m$ sideband region, $0.165 \text{ GeV} \leq \delta m \leq 0.171 \text{ GeV}$, where the number of real $D^+$'s is relatively small. Fig. E.5 shows this comparison of the $M_{K^-\pi^+\pi^+}$ plot from CLEO II.V (scaled with luminosity) with that of CLEO II. The ratio of the areas of the plots is $1.061 \pm 0.013$, smaller than the expected value. However, this 6.1% excess of the area in the CLEO II.V plot is comparable with the excess of the CLEO II.V (scaled) raw yield in $D^+ \rightarrow K^-\pi^+\pi^+$ mode. Since the measured excess is much smaller than the expected, it is difficult to conclude that the excess in CLEO II.V was made by the difference in the numbers of $\pi^0$'s and tracks between CLEO II.V and CLEO II. Nevertheless, the possibility of systematic error in the analysis results due to the difference in the numbers of $\pi^0$'s and tracks between CLEO II.V and CLEO II can not be excluded.

The comparison of the numbers of entries in $M_{K^-\pi^+}$ plots from the CLEO II.V and CLEO II data in the sideband region of $\delta m$, $0.195 \text{ GeV} \leq \delta m \leq 0.215 \text{ GeV}$, was
executed to check whether the raw yield difference between CLEO II.V and CLEO II $D^+ \rightarrow \bar{K}^{*0} e^+ \nu_e$ analyses was made by statistical fluctuations. The number of entries of the scaled CLEO II.V $M_{K^{-}\pi^+}$ plot was 6779.5 ± 60.5359 and that of the CLEO II $M_{K^{-}\pi^+}$ plot was 5239.0 ± 72.381. The expected ratio of the background amount in the scaled CLEO II.V $M_{K^{-}\pi^+}$ plot with respect to that of CLEO II is 1.075 ± 0.006 which is the same in the inclusive $M_{K\pi}$ case. However, if the difference of the numbers of $\pi^0$s between CLEO II.V and CLEO II is multiplied, the background ratio can be up to 1.120 ± 0.007. The effect of the difference in the number of $\pi^0$s must be accounted for because the $M_{K^{-}\pi^+}$ plots were made with ntuples produced with possible combinations of $K^-, \pi^+, e^+$ and $\pi^0$. The ratio of the background amounts in $M_{K^{-}\pi^+}$ plots was 1.294±0.021, a deviation from 1.0 that is about 2.5 times larger than expected. However, this result implies that the lower (scaled) raw yield of CLEO II.V compared to that of CLEO II in the $D^+ \rightarrow \bar{K}^{*0} e^+ \nu_e$ mode is not due to missing some CLEO II.V data or analyzing some part of CLEO II data multiple times. Fig. E.6 shows the comparison of the $M_{K^{-}\pi^+}$ plot from CLEO II.V (scaled with luminosity) with that of CLEO II in the $D^+ \rightarrow \bar{K}^{*0} e^+ \nu_e$ mode in 0.195 GeV ≤ $\delta m$ ≤ 0.215 GeV.
Figure E.6: Comparison of the $M_{K^{-}π^{+}}$ plot from CLEO II.V (scaled with luminosity) with that of CLEO II in the $D^{+} \rightarrow \bar{K}^{*0} e^{+} \nu_e$ mode in $δm$ sideband region $0.195 \text{ GeV} \leq δm \leq 0.215 \text{ GeV}$.

As the final check in the study of the difference in $R_e$'s between CLEO II.V and CLEO II, I studied whether or not the “$ν$ reconstruction” method used in the $D^{+} \rightarrow \bar{K}^{*0} l^{+} \nu_l$ analysis introduces some bias to the analysis results. As described in section 6.9, the “$ν$ reconstruction” method was tested with CLEO II.V data. This test was repeated with CLEO II data. Results of the $δm$ fit using the full reconstruction and “$ν$ reconstruction” methods are shown in Fig. 6.12 for CLEO II.V and in Fig. E.7 for CLEO II data. Table E.4 summarizes the results of this study. As shown in Table E.4, the results with the full reconstruction and those with “$ν$ reconstruction” agree well. This implies that the “$ν$ reconstruction” method retains the analysis results.
Figure E.7: $\delta m$ fits in CLEO II $D^+ \to K^- \pi^+ \pi^+$ analysis with the full reconstruction (left) and the “$\nu$ reconstruction” method (right).

Table E.4: Summary of the results of “$\nu$ reconstruction” method check with $D^+ \to K^- \pi^+ \pi^+$ mode. In the table, $N$, $\epsilon$ and $N_\epsilon$ indicate raw yield, efficiency and efficiency-corrected yield, respectively.

<table>
<thead>
<tr>
<th></th>
<th>CLEO II.V</th>
<th></th>
<th>CLEO II</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Full Recon</td>
<td>$\nu$ Recon</td>
<td>Full Recon</td>
<td>$\nu$ Recon</td>
</tr>
<tr>
<td>$N$</td>
<td>25246.0 ± 406.3</td>
<td>24726.0 ± 905.8</td>
<td>12367.0 ± 287.7</td>
<td>12359.0 ± 813.4</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>0.156</td>
<td>0.149</td>
<td>0.158</td>
<td>0.149</td>
</tr>
<tr>
<td>$N_\epsilon$</td>
<td>161649.1 ± 2601.7</td>
<td>165963.1 ± 6079.9</td>
<td>78148.3 ± 1818.0</td>
<td>83060.5 ± 5466.7</td>
</tr>
<tr>
<td>$(N_\epsilon)<em>\nu/(N</em>\epsilon)_{full}$</td>
<td>0.974 ± 0.051</td>
<td></td>
<td>0.941 ± 0.084</td>
<td></td>
</tr>
</tbody>
</table>
REFERENCES


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